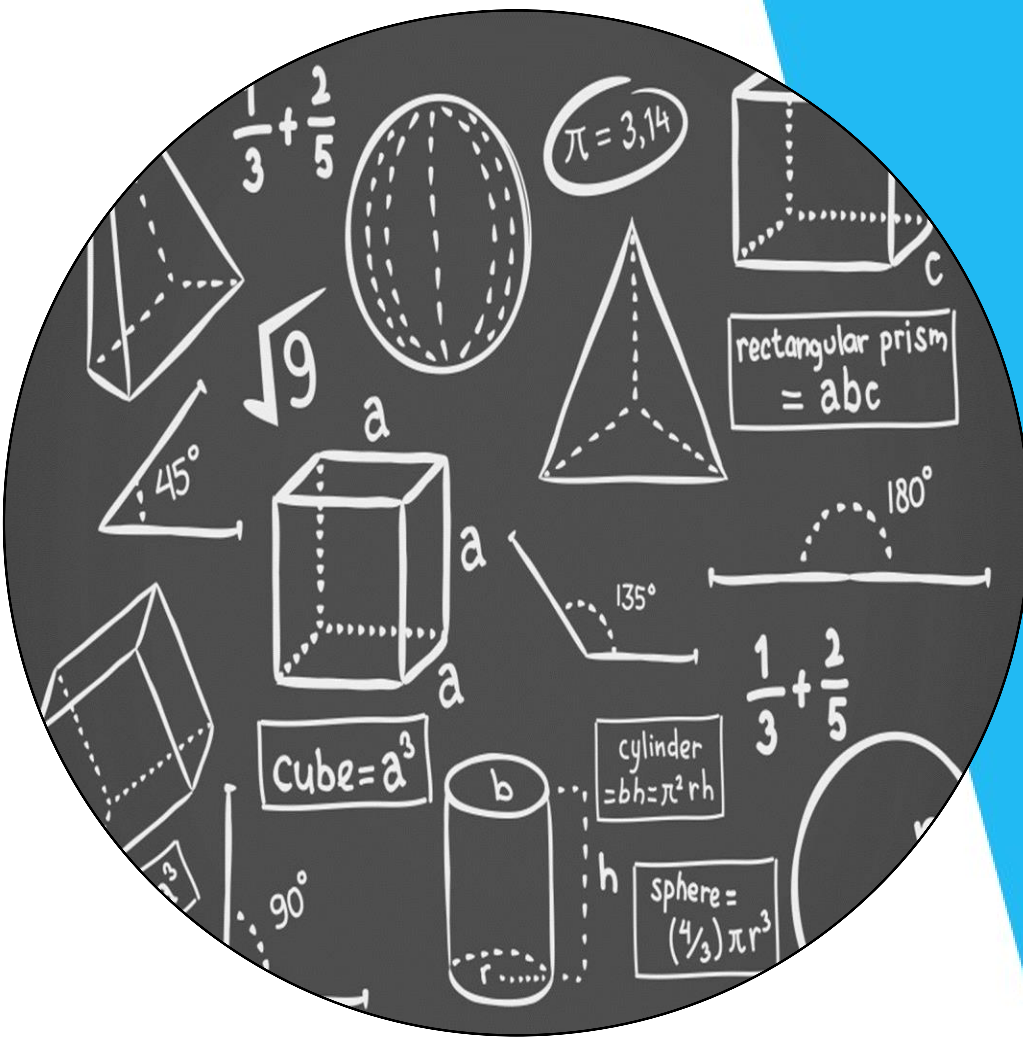
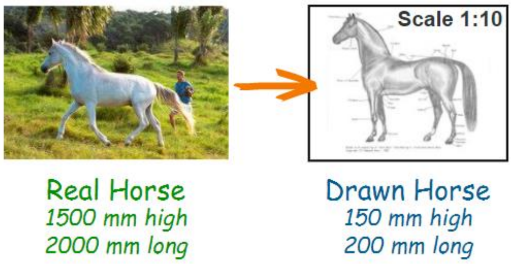
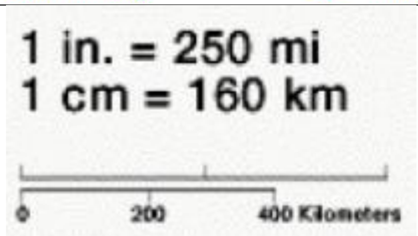
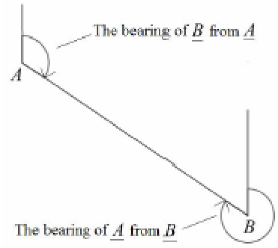
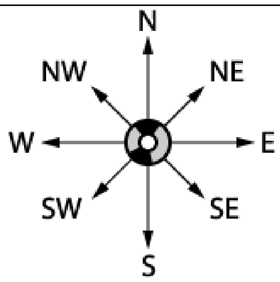
Knowledge Organiser



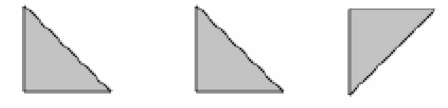
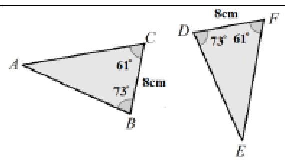
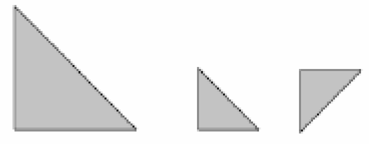
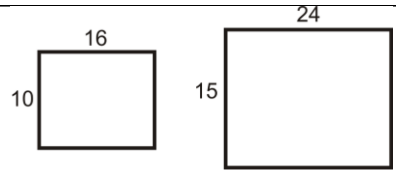
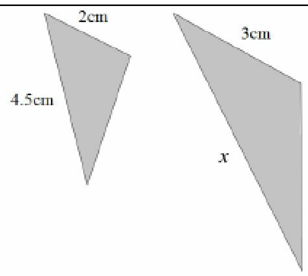
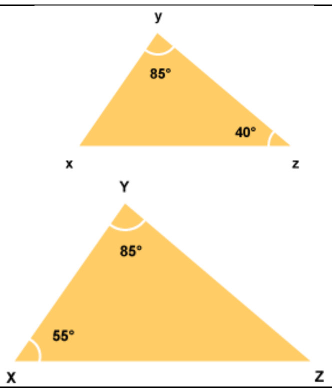
Mathematics – 11x1

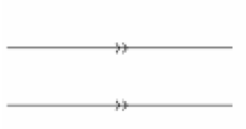
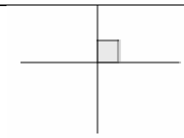
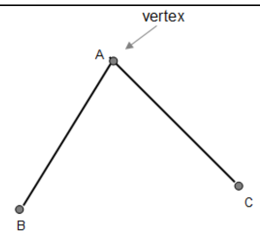
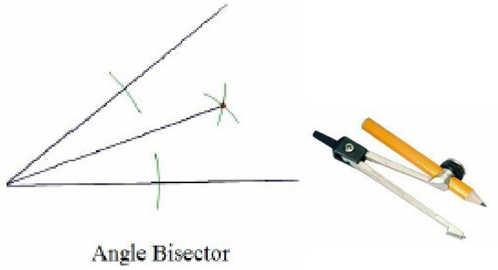
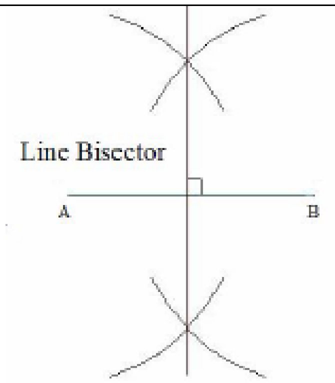
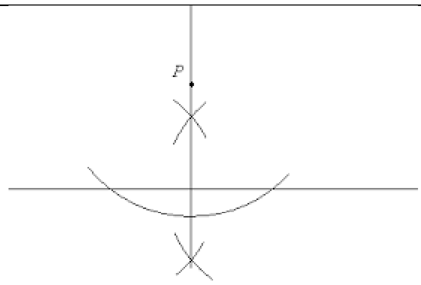
RAYNES
PARK HIGH SCHOOL

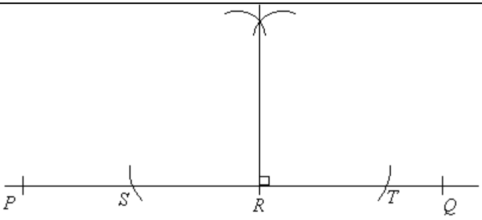
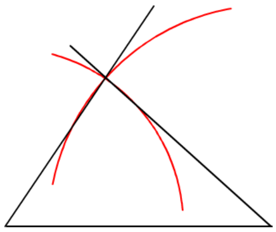
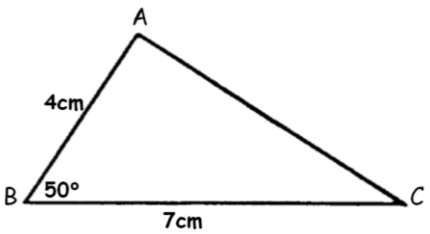
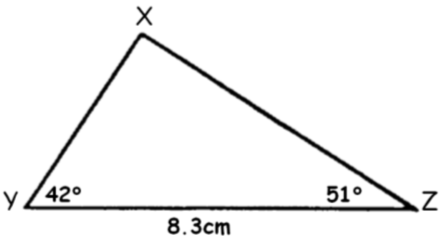
Topic: Bearings and Scale Diagrams

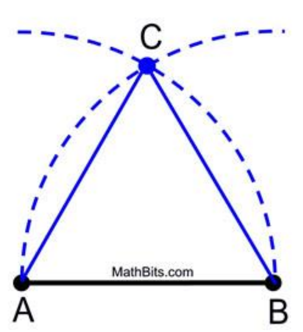
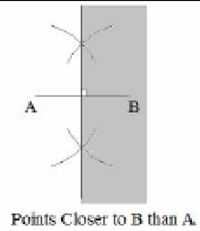
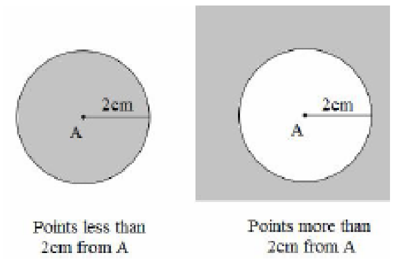
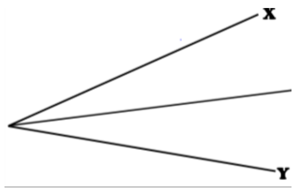
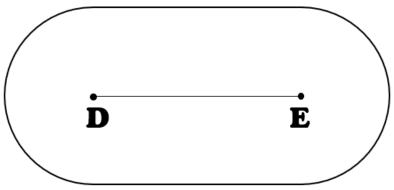
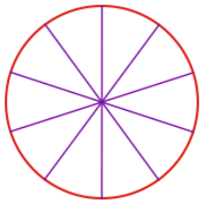
Topic/Skill	Definition/Tips	Example
1. Scale	The **ratio** of the **length** in a **model** to the length of the **real** thing.	
2. Scale (Map)	The **ratio** of a **distance on the map** to the actual **distance in real life**.	
3. Bearings	1. Measure from **North** (draw a North line) 2. Measure **clockwise** 3. Your answer must have **3 digits** (eg. 047°) Look out for where the bearing is measured <u>from</u>.	
4. Compass Directions	You can use an acronym such as **'Never Eat Shredded Wheat'** to remember the order of the compass directions in a clockwise direction. Bearings: $NE = 045°, W = 270°\ etc.$	

Topic: Congruence and Similarity

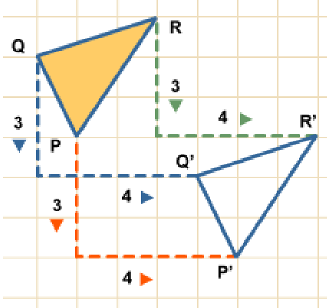
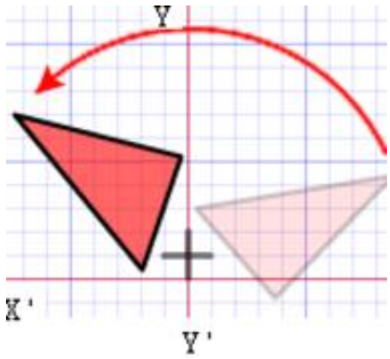
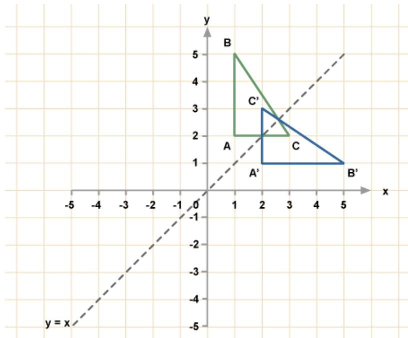
Topic/Skill	Definition/Tips	Example
1. Congruent Shapes	Shapes are congruent if they are **identical - same shape** and **same size.** Shapes can be rotated or reflected but still be congruent.	
2. Congruent Triangles	4 ways of proving that two triangles are congruent: 1. **SSS** (Side, Side, Side) 2. **RHS** (Right angle, Hypotenuse, Side) 3. **SAS** (Side, Angle, Side) 4. **ASA** (Angle, Side, Angle) or **AAS** <u>ASS does not prove congruency.</u>	 $BC = DF$ $\angle ABC = \angle EDF$ $\angle ACB = \angle EFD$ ∴ The two triangles are congruent by AAS.
3. Similar Shapes	Shapes are similar if they are the **same shape but different sizes**. The proportion of the matching sides must be the same, meaning the ratios of corresponding sides are all equal.	
4. Scale Factor	The **ratio of corresponding sides** of two similar shapes. To find a scale factor, **divide a length** on one shape **by the corresponding length** on a similar shape.	 Scale Factor = 15 ÷ 10 = 1.5
5. Finding missing lengths in similar shapes	1. Find the **scale factor**. 2. **Multiply or divide** the corresponding side to find a missing length. If you are finding a missing length on the larger shape you will need to multiply by the scale factor. If you are finding a missing length on the smaller shape you will need to divide by the scale factor.	 Scale Factor = 3 ÷ 2 = 1.5 $x = 4.5 \times 1.5 = 6.75cm$
6. Similar Triangles	To show that two triangles are similar, show that: 1. The three sides are in the same proportion 2. Two sides are in the same proportion, and their included angle is the same 3. The three angles are equal	

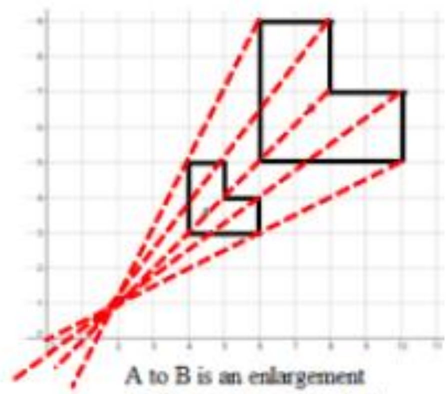
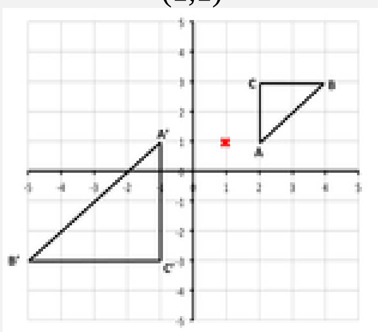
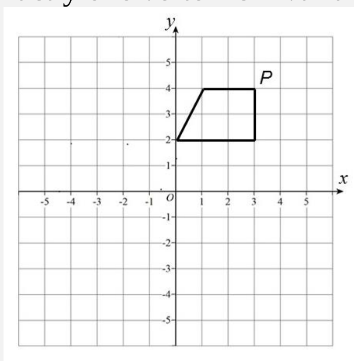
Topic/Skill	Definition/Tips	Example
1. Parallel	Parallel lines never meet.	
2. Perpendicular	Perpendicular lines are at right angles. There is a 90° angle between them.	
3. Vertex	A corner or a point where two lines meet.	
4. Angle Bisector	**Angle Bisector: Cuts the angle in half**. 1. Place the sharp end of a pair of compasses on the vertex. 2. Draw an arc, marking a point on each line. 3. Without changing the compass put the compass on each point and mark a centre point where two arcs cross over. 4. Use a ruler to draw a line through the vertex and centre point.	
5. Perpendicular Bisector	**Perpendicular Bisector: Cuts a line in half and at right angles**. 1. Put the sharp point of a pair of compasses on A. 2. Open the compass over half way on the line. 3. Draw an arc above and below the line. 4. Without changing the compass, repeat from point B. 5. Draw a straight line through the two intersecting arcs.	
6. Perpendicular from an External Point	The **perpendicular distance** from a point to a line is the **shortest distance** to that line. 1. Put the sharp point of a pair of compasses on the point. 2. Draw an arc that crosses the line twice. 3. Place the sharp point of the compass on one of these points, open over half way and draw an arc above and below the line. 4. Repeat from the other point on the line.	

	5. Draw a straight line through the two intersecting arcs.	
7. Perpendicular from a Point on a Line	Given line PQ and point R on the line: 1. Put the sharp point of a pair of compasses on point R. 2. Draw two arcs either side of the point of equal width (giving points S and T) 3. Place the compass on point S, open over halfway and draw an arc above the line. 4. Repeat from the other arc on the line (point T). 5. Draw a straight line from the intersecting arcs to the original point on the line.	
8. Constructing Triangles (Side, Side, Side)	1. Draw the base of the triangle using a ruler. 2. Open a pair of compasses to the width of one side of the triangle. 3. Place the point on one end of the line and draw an arc. 4. Repeat for the other side of the triangle at the other end of the line. 5. Using a ruler, draw lines connecting the ends of the base of the triangle to the point where the arcs intersect.	
9. Constructing Triangles (Side, Angle, Side)	1. Draw the base of the triangle using a ruler. 2. Measure the angle required using a protractor and mark this angle. 3. Remove the protractor and draw a line of the exact length required in line with the angle mark drawn. 4. Connect the end of this line to the other end of the base of the triangle.	
10. Constructing Triangles (Angle, Side, Angle)	1. Draw the base of the triangle using a ruler. 2. Measure one of the angles required using a protractor and mark this angle. 3. Draw a straight line through this point from the same point on the base of the triangle. 4. Repeat this for the other angle on the other end of the base of the triangle.	

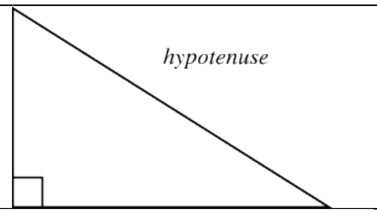
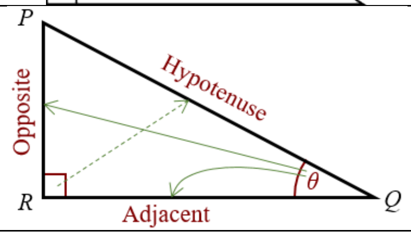
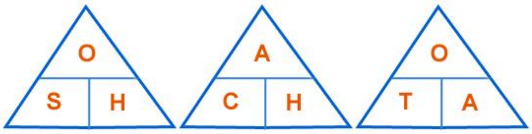
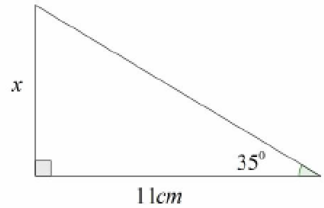
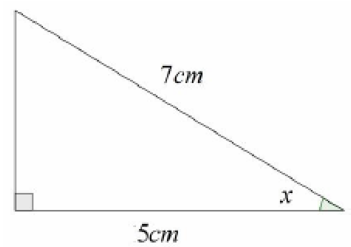
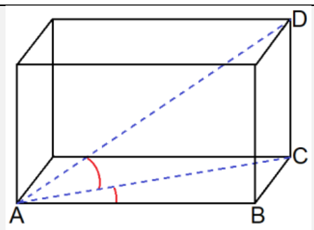
11. Constructing an Equilateral Triangle (also makes a 60° angle)	1. Draw the base of the triangle using a ruler. 2. Open the pair of compasses to the exact length of the side of the triangle. 3. Place the sharp point on one end of the line and draw an arc. 4. Repeat this from the other end of the line. 5. Using a ruler, draw lines connecting the ends of the base of the triangle to the point where the arcs intersect.	
12. Loci and Regions	A **locus** is a **path of points that follow a rule**. For the locus of points **closer to B than A**, create a **perpendicular bisector** between A and B and shade the side closer to B. For the locus of points **equidistant from A**, use a compass to draw a **circle**, centre A. For the locus of points **equidistant to line X and line Y**, create an **angle bisector**. For the locus of points a set **distance from a line**, create **two semi-circles** at either end joined by **two parallel lines**.	   
13. Equidistant	A point is equidistant from a set of objects if the **distances between that point and each of the objects is the same**.	

Topic: Shape Transformations

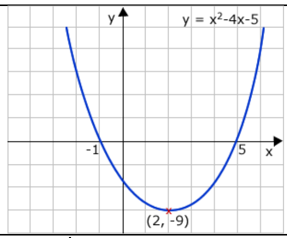
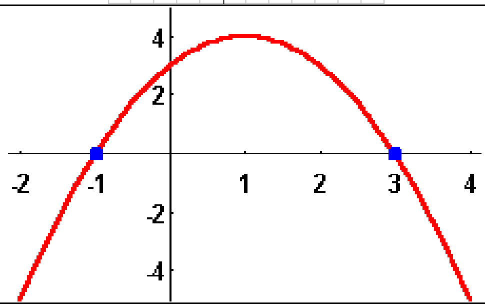
Topic/Skill	Definition/Tips	Example
1. Translation	**Translate** means to **move a shape**. The shape does not change **size** or **orientation**.	
2. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix}2\\3\end{pmatrix}$ means '2 right, 3 up' $\begin{pmatrix}-1\\-5\end{pmatrix}$ means '1 left, 5 down'
3. Rotation	The size does not change, but the **shape is turned around a point**. Use tracing paper.	Rotate Shape A 90° anti-clockwise about (0,1) 
4. Reflection	The size does not change, but the shape is '**flipped**' like in a **mirror**. Line x =? is a **vertical line**. Line y =? is a **horizontal line**. Line $y = x$ is a **diagonal line**.	Reflect shape C in the line $y = x$ 
5. Enlargement	The shape will get **bigger or smaller**. Multiply each side by the **scale factor**.	Scale Factor = 3 means '3 times larger = multiply by 3' Scale Factor = ½ means 'half the size = divide by 2'

6. Finding the Centre of Enlargement	Draw **straight lines** through **corresponding corners** of the two shapes. The centre of enlargement is the point **where all the lines cross over**. Be careful with negative enlargements as the corresponding corners will be the other way around.	 A to B is an enlargement SF 2 about the point (2,1)
7. Describing Transformations	Give the following information when describing each transformation: Look at the number of marks in the question for a hint of how many pieces of information are needed. If you are asked to describe a 'transformation', you need to say the **name of the type of transformation** as well as the other details.	**- Translation, Vector** **- Rotation, Direction, Angle, Centre** **- Reflection, Equation of mirror line** **- Enlargement, Scale factor, Centre of enlargement**
8. Negative Scale Factor Enlargements	Negative enlargements will **look like they have been rotated**. $SF = -2$ will be rotated, and also twice as big.	Enlarge ABC by scale factor -2, centre (1,1) 
9. Invariance	A point, line or shape is invariant if it **does not change/move** when a transformation is performed. An invariant point 'does not vary'.	If shape P is reflected in the $y - axis$, then exactly one vertex is invariant. 

Topic: Right Angled Trigonometry

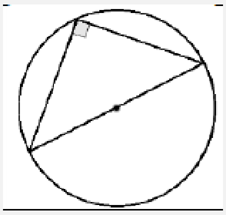
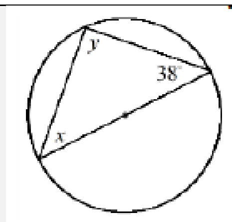
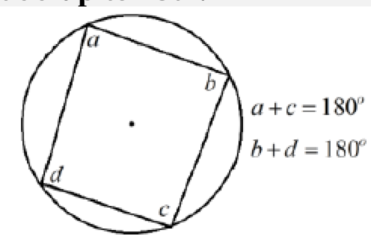
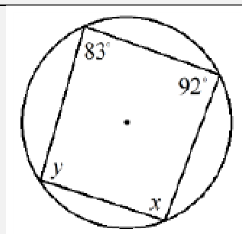
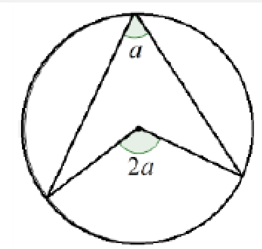
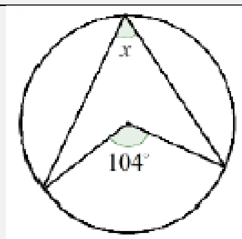
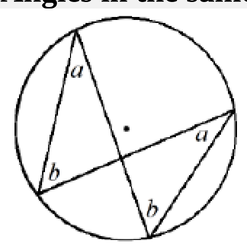
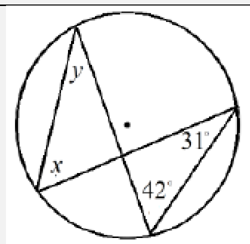
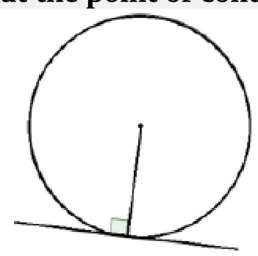
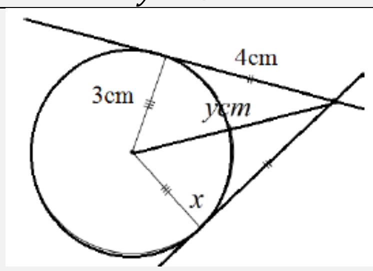
Topic/Skill	**Definition/Tips**	**Example**
1. Trigonometry	The **study** of **triangles**.	
2. Hypotenuse	The **longest side** of a **right-angled triangle**. Is always **opposite** the **right angle**.	
3. Adjacent	**Next to**	
4. Trigonometric Formulae	Use **SOHCAHTOA**. $\boldsymbol{\sin\theta = \frac{O}{H}}$ $\boldsymbol{\cos\theta = \frac{A}{H}}$ $\boldsymbol{\tan\theta = \frac{O}{A}}$  When finding a missing angle, use the 'inverse' trigonometric function by pressing the 'shift' button on the calculator.	 Use 'Opposite' and 'Adjacent', so use 'tan' $\tan 35 = \frac{x}{11}$ $x = 11\tan 35 = 7.70cm$  Use 'Adjacent' and 'Hypotenuse', so use 'cos' $\cos x = \frac{5}{7}$ $x = cos^{-1}\left(\frac{5}{7}\right) = 44.4°$
5. 3D Trigonometry	Find missing lengths by **identifying right angled triangles**. You will often have to find a missing length you are not asked for before finding the missing length you are asked for.	

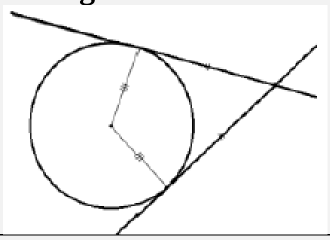
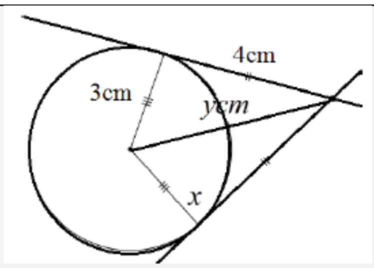
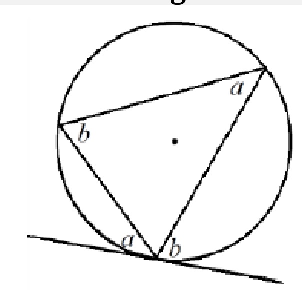
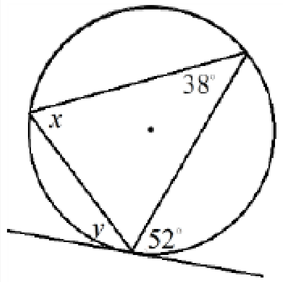
Topic: Further Quadratics

Topic/Skill	**Definition/Tips**	**Example**
1. Quadratic	A quadratic expression is of the form $\boldsymbol{ax^2 + bx + c}$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions: $2x^3 - 5x^2$ $9x - 1$
2. Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2 - b^2}$ can be factorised to give $(\boldsymbol{a + b})(\boldsymbol{a - b})$	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
4. Solving Quadratics $(ax^2 = b)$	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
5. Solving Quadratics $(ax^2 + bx = 0)$	**Factorise** and then **solve = 0**.	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0 \ or \ x = 3$
6. Solving Quadratics by Factorising $(a = 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x + 5)(x - 2) = 0$ $x = -5 \ or \ x = 2$
7. Quadratic Graph	A '**U-shaped**' curve called a **parabola**. The equation is of the form $y = \boldsymbol{ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0}$**,** the parabola is **upside down**.	
8. Roots of a Quadratic	A root is a **solution**. The roots of a quadratic are the **$\boldsymbol{x}$-intercepts of the quadratic graph**.	

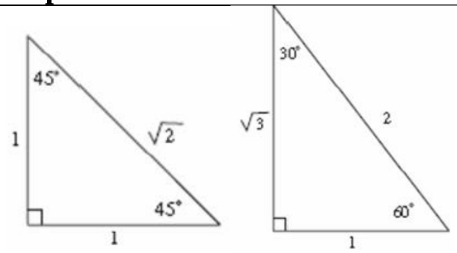
9. Turning Point of a Quadratic	A turning point is the **point where a quadratic turns**. On a **positive parabola**, the turning point is called a **minimum**. On a **negative parabola**, the turning point is called a **maximum**.	
10. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
11. Solving Quadratics by Factorising $(a \neq 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2}\ or\ x = -4$
12. Completing the Square (when $a = 1$)	A quadratic in the form $x^2 + bx + c$ can be written in the form $(\boldsymbol{x} + \boldsymbol{p})^2 + \boldsymbol{q}$ 1. Write a set of brackets with x in and **half** the value of b. 2. Square the bracket. 3. Subtract $\left(\frac{b}{2}\right)^2$ and add c. 4. Simplify the expression. You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph.	Complete the square of $y = x^2 - 6x + 2$ Answer: $(x - 3)^2 - 3^2 + 2$ $= (x - 3)^2 - 7$ The minimum value of this expression occurs when $(x - 3)^2 = 0$, which occurs when $x = 3$ When $x = 3$, $y = 0 - 7 = -7$ Minimum point = $(3, -7)$
13. Completing the Square (when $a \neq 1$)	A quadratic in the form $ax^2 + bx + c$ can be written in the form $\mathbf{p}(\boldsymbol{x} + \boldsymbol{q})^2 + \boldsymbol{r}$ Use the same method as above, but factorise out a at the start.	Complete the square of $4x^2 + 8x - 3$ Answer: $4[x^2 + 2x] - 3$ $= 4[(x + 1)^2 - 1^2] - 3$ $= 4(x + 1)^2 - 4 - 3$ $= 4(x + 1)^2 - 7$
14. Solving Quadratics by Completing the Square	**Complete the square** in the usual way and **use inverse operations to solve**.	Solve $x^2 + 8x + 1 = 0$ Answer: $(x + 4)^2 - 4^2 + 1 = 0$ $(x + 4)^2 - 15 = 0$

		$(x+4)^2 = 15$ $(x+4) = \pm\sqrt{15}$ $x = -4 \pm \sqrt{15}$
15. Solving Quadratics using the Quadratic Formula	A quadratic in the form $ax^2 + bx + c = 0$ can be solved using the formula: $\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ Use the formula if the quadratic does not factorise easily.	Solve $3x^2 + x - 5 = 0$ Answer: $a = 3, b = 1, c = -5$ $x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$ $x = \frac{-1 \pm \sqrt{61}}{6}$ $x = 1.14 \; or -1.47 \; (2 \; d.p.)$

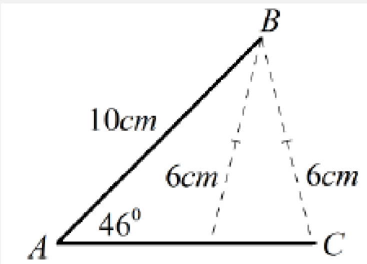
Topic/Skill	Definition/Tips	Example
Circle Theorem 1	**Angles in a semi-circle have a right angle at the circumference.** 	 $y = 90°$ $x = 180 - 90 - 38 = 52°$
Circle Theorem 2	**Opposite angles in a cyclic quadrilateral add up to 180°.** 	 $x = 180 - 83 = 97°$ $y = 180 - 92 = 88°$
Circle Theorem 3	**The angle at the centre is twice the angle at the circumference.** 	 $x = 104 \div 2 = 52°$
Circle Theorem 4	**Angles in the same segment are equal.** 	 $x = 42°$ $y = 31°$
Circle Theorem 5	**A tangent is perpendicular to the radius at the point of contact.** 	 $y = 5cm$ (Pythagoras' Theorem)

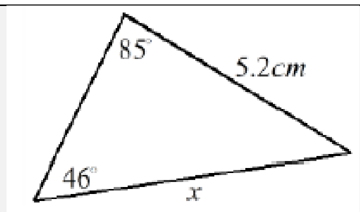
Circle Theorem 6	**Tangents from an external point at equal in length.** 	 $x = 90°$
Circle Theorem 7	**Alternate Segment Theorem** 	 $x = 52°$ $y = 38°$

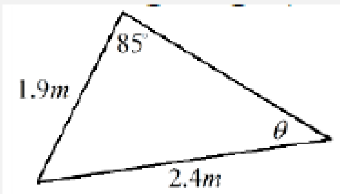
Topic: Trigonometry

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Exact Values for Angles in Trigonometry</td><td>
<table>
<tr><th></th><th>0°</th><th>30°</th><th>45°</th><th>60°</th><th>90°</th></tr>
<tr><td>sin</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{\sqrt{3}}{2}$</td><td>1</td></tr>
<tr><td>cos</td><td>1</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>tan</td><td>0</td><td>$\frac{1}{\sqrt{3}}$</td><td>1</td><td>$\sqrt{3}$</td><td>----</td></tr>
</table>
</td><td>


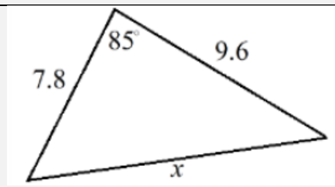
</td></tr>
<tr><td>2. Sine Rule</td><td>
Use with non right angle triangles.
Use when the question involves 2 sides and 2 angles.

For missing side:
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
For missing angle:
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
There is an ambiguous case (where there are two potential answers)


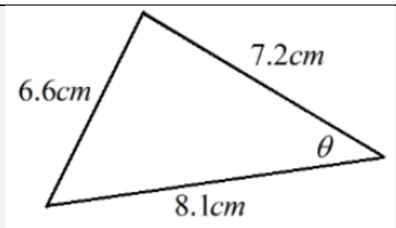
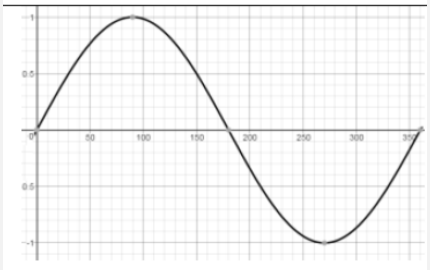
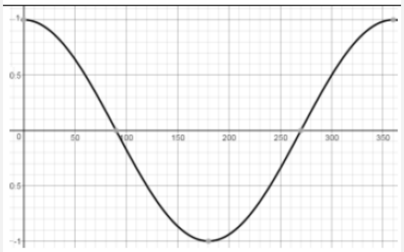
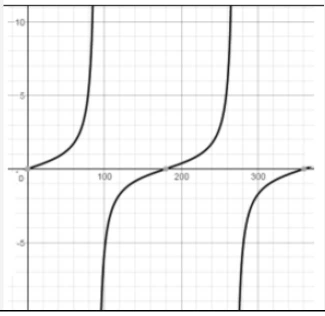
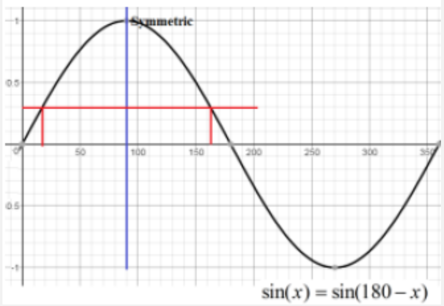
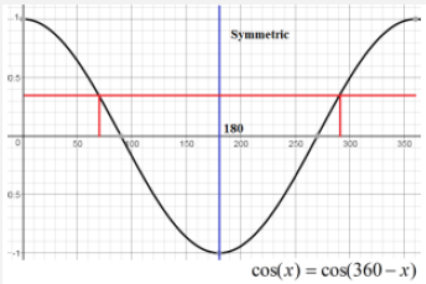
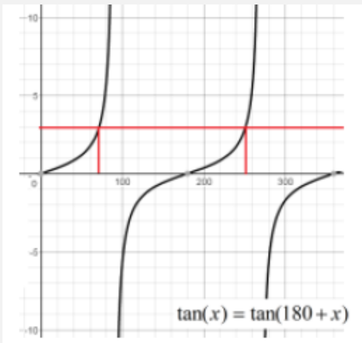
To find the two angles, use sine to find one, and then subtract your answer from 180 to find the other answer.
</td><td>


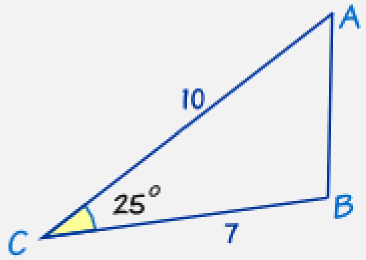
$$\frac{x}{\sin 85} = \frac{5.2}{\sin 46}$$
$$x = \frac{5.2 \times \sin 85}{\sin 46} = 3.75cm$$


$$\frac{\sin\theta}{1.9} = \frac{\sin 85}{2.4}$$
$$\sin\theta = \frac{1.9 \times \sin 85}{2.4} = 0.789$$
$$\theta = sin^{-1}(0.789) = 52.1°$$
</td></tr>
<tr><td>3. Cosine Rule</td><td>
Use with non right angle triangles.
Use when the question involves 3 sides and 1 angle.

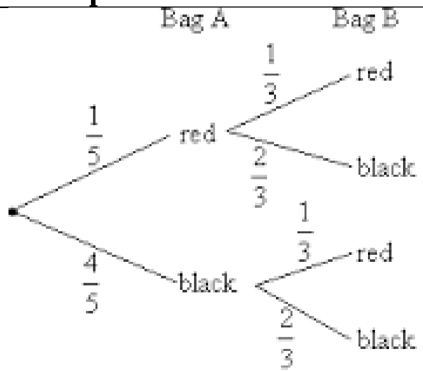
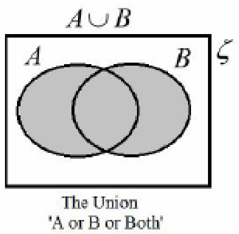
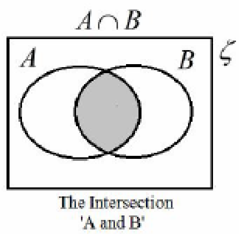
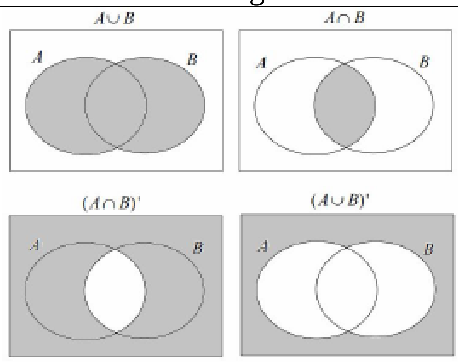
For missing side:
$$a^2 = b^2 + c^2 - 2bccosA$$
For missing angle:
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
</td><td>


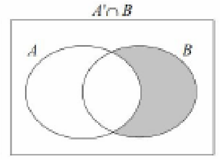
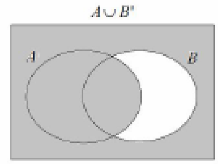
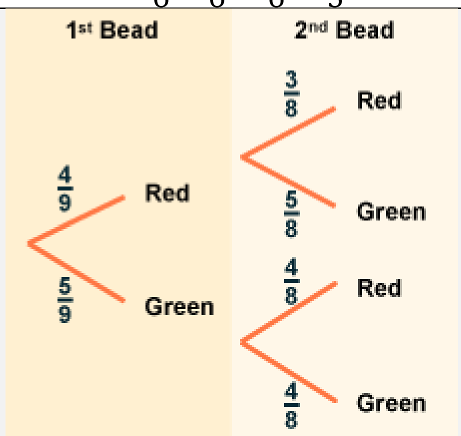
$$x^2 = 9.6^2 + 7.8^2 - (2 \times 9.6 \times 7.8 \times \cos 85)$$
$$x = 11.8$$
</td></tr>
</table>

		 $$\cos\theta = \frac{7.2^2 + 8.1^2 - 6.6^2}{2 \times 7.2 \times 8.1}$$ $$\theta = 50.7°$$
4. Graphs of Trigonometric Functions	$y = \sin(x)$ for $0 \le x \le 360^o$  $y = \cos(x)$ for $0 \le x \le 360^o$  $y = \tan(x)$ for $0 \le x \le 360^o$ 	  

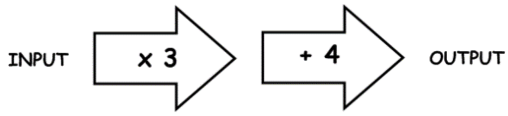
5. Area of a Triangle	Use when given the **length of two sides and the included angle**. $$\boldsymbol{Area\ of\ a\ Triangle = \frac{1}{2}ab\sin C}$$	 $$A = \frac{1}{2}ab\sin C$$ $$A = \frac{1}{2} \times 7 \times 10 \times \sin 25$$ $$A = 14.8$$

Topic: Probability (Trees and Venns)

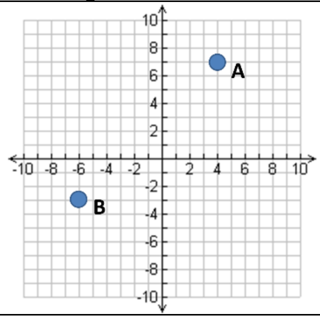
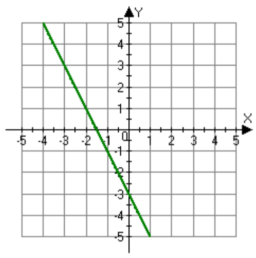
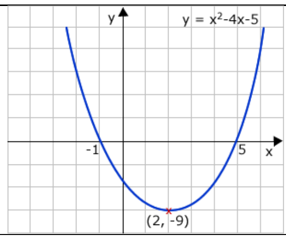
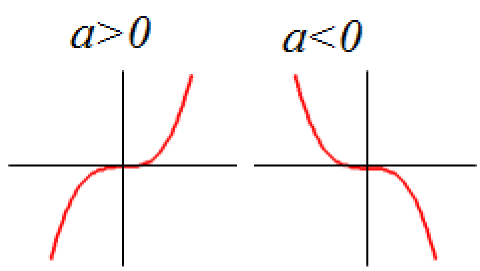
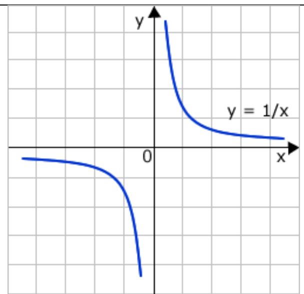
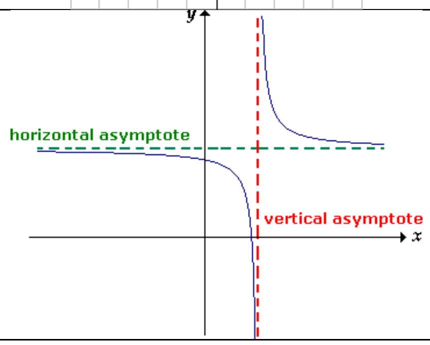
Topic/Skill	Definition/Tips	Example
1. Tree Diagrams	Tree diagrams show **all the possible outcomes** of an event and calculate their probabilities. **All branches must add up to 1 when adding downwards.** This is because the **probability of something not happening** is **1 minus the probability that it does happen**. **Multiply** going **across** a tree diagram. **Add** going **down** a tree diagram.	
2. Independent Events	The outcome of a **previous event does not influence/affect the outcome of a second event**.	An example of independent events could be <u>replacing</u> a counter in a bag after picking it.
3. Dependent Events	The outcome of a **previous event does influence/affect the outcome of a second event**.	An example of dependent events could be not replacing a counter in a bag after picking it. '<u>Without replacement</u>'
4. Probability Notation	**P(A)** refers to the **probability that event A will occur**. **P(A')** refers to the **probability that event A will <u>not</u> occur**. **P(A ∪ B)** refers to the **probability that event A <u>or</u> B <u>or</u> both will occur.** **P(A ∩ B)** refers to the **probability that <u>both</u> events A and B will occur.**	P(Red Queen) refers to the probability of picking a Red Queen from a pack of cards. P(Blue') refers to the probability that you do not pick Blue. P(Blonde ∪ Right Handed) refers to the probability that you pick someone who is Blonde or Right Handed or both. P(Blonde ∩ Right Handed) refers to the probability that you pick someone who is both Blonde and Right Handed.
5. Venn Diagrams	A Venn Diagram shows the **relationship between a group of different things** and how they overlap. You may be asked to shade Venn Diagrams as shown below and to the right.  	

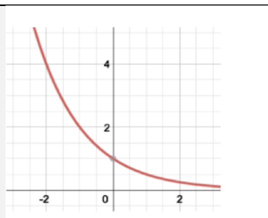
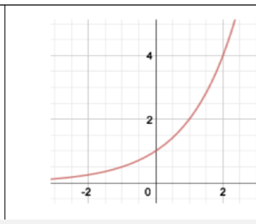
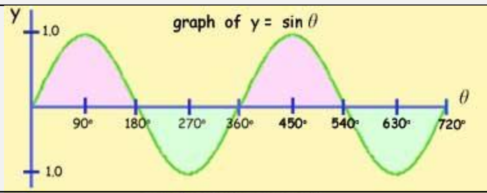
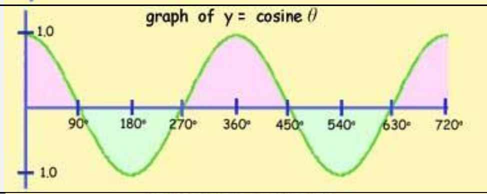
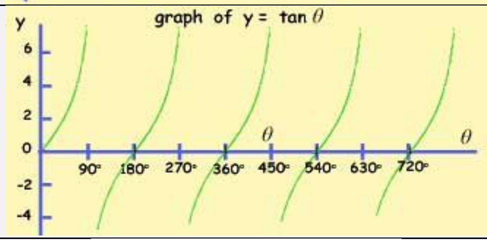
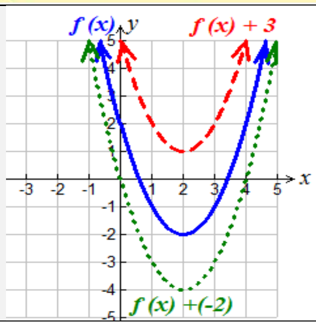
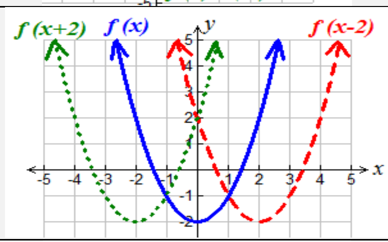
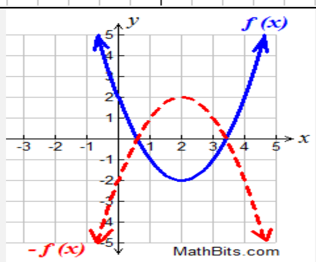
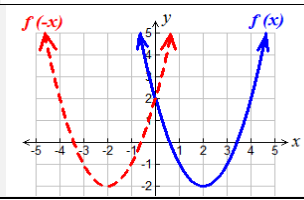
		 
6. Venn Diagram Notation	∈ means '**element of a set**' (a value in the set) { } means the collection of values in the set. ξ means the '**universal set**' (all the values to consider in the question) **A' means 'not in set A' (called complement)** **A ∪ B means 'A or B or both' (called Union)** **A ∩ B means 'A and B (called Intersection)**	Set A is the even numbers less than 10. A = {2, 4, 6, 8} Set B is the prime numbers less than 10. B = {2, 3, 5, 7} A ∪ B = {2, 3, 4, 5, 6, 7, 8} A ∩ B = {2}
7. AND rule for Probability	When two events, A and B, are **independent**: $\boldsymbol{P(A \text{ and } B) = P(A) \times P(B)}$	What is the probability of rolling a 4 and flipping a Tails? $P(4 \text{ and } Tails) = P(4) \times P(Tails)$ $= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
8. OR rule for Probability	When two events, A and B, are **mutually exclusive**: $\boldsymbol{P(A \text{ or } B) = P(A) + P(B)}$	What is the probability of rolling a 2 or rolling a 5? $P(2 \text{ or } 5) = P(2) + P(5)$ $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$
9. Conditional Probability	The probability of an event A happening, **given that** event B has already happened. With conditional probability, check if the numbers on the second branches of a tree diagram changes. For example, if you have 4 red beads in a bag of 9 beads and pick a red bead on the first pick, then there will be 3 red beads left out of 8 beads on the second pick.	

Subject: Maths **Topic: Functions**

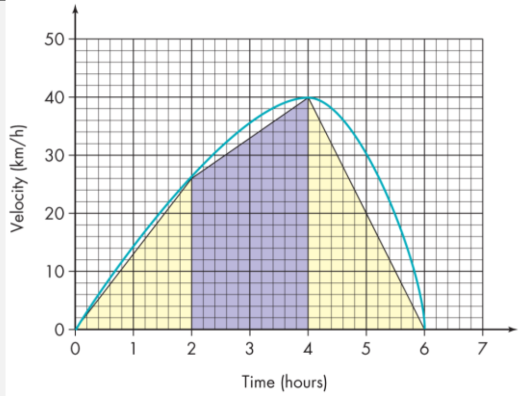
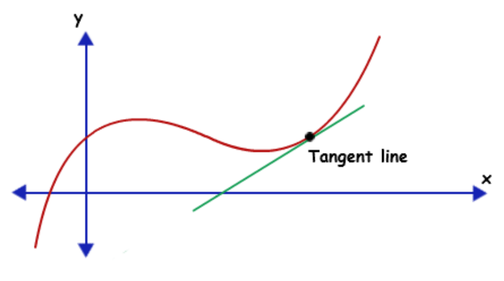
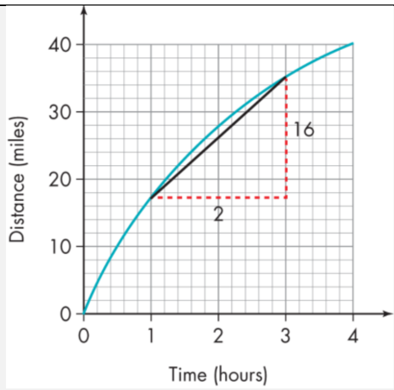
Topic/Skill	Definition/Tips	Example
1. Function Machine	Takes an **input** value, performs some **operations** and produces an **output** value.	
2. Function	A **relationship** between two sets of values.	$f(x) = 3x^2 - 5$ 'For any input value, square the term, then multiply by 3, then subtract 5'.
3. Function notation	$f(x)$ $\boldsymbol{x}$ is the **input** value $\boldsymbol{f(x)}$ is the **output** value.	$f(x) = 3x + 11$ Suppose the input value is $x = 5$ The output value is $f(5) = 3 \times 5 + 11 = 26$
4. Inverse function	$f^{-1}(x)$ A function that performs the **opposite process** of the original function. 1. Write the function as $y = f(x)$ 2. Rearrange to make x the subject. 3. Replace the $\boldsymbol{y}$ **with** $\boldsymbol{x}$ and the $\boldsymbol{x}$ **with** $\boldsymbol{f^{-1}(x)}$	$f(x) = (1 - 2x)^5$. Find the inverse. $y = (1 - 2x)^5$ $\sqrt[5]{y} = 1 - 2x$ $1 - \sqrt[5]{y} = 2x$ $\frac{1-\sqrt[5]{y}}{2} = x$ $f^{-1}(x) = \dfrac{1 - \sqrt[5]{x}}{2}$
5. Composite function	A **combination** of two or more **functions** to create a new function. $\boldsymbol{fg(x)}$ is the composite function that **substitutes** the function $\boldsymbol{g(x)}$ **into** the function $\boldsymbol{f(x)}$. $\boldsymbol{fg(x)}$ means '**do g first, then f**' $\boldsymbol{gf(x)}$ means '**do f first, then g**'	$f(x) = 5x - 3$, $g(x) = \frac{1}{2}x + 1$ What is $fg(4)$? $g(4) = \frac{1}{2} \times 4 + 1 = 3$ $f(3) = 5 \times 3 - 3 = 12 = fg(4)$ What is $fg(x)$? $fg(x) = 5\left(\frac{1}{2}x + 1\right) - 3 = \frac{5}{2}x + 2$

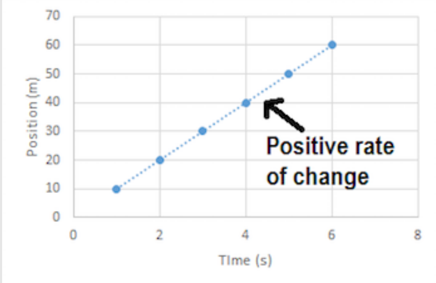
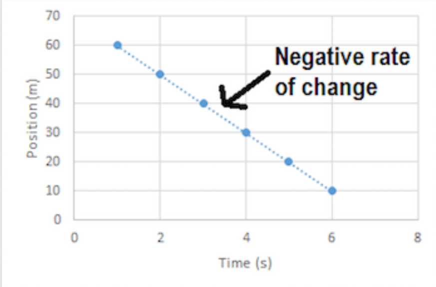
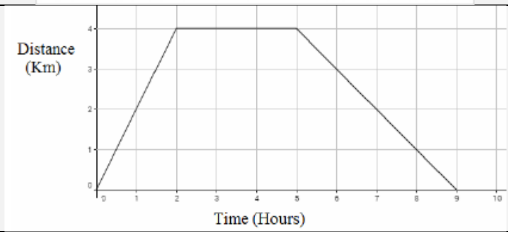
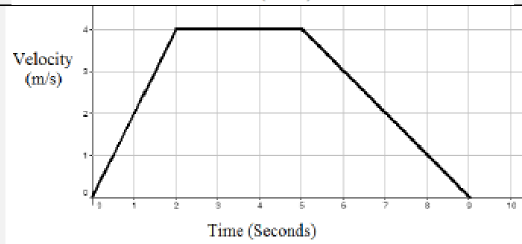
Subject: Maths **Topic: Graphs and Graph Transformations**

Topic/Skill	Definition/Tips	Example
1. Coordinates	Written in **pairs**. The **first** term is the **x-coordinate** (movement **across**). The **second** term is the **y-coordinate** (movement **up or down**)	 A: (4,7) B: (-6,-3)
2. Linear Graph	**Straight line** graph. The **equation** of a linear graph can contain an **x-term**, a **y-term** and a **number**.	Example:  Other examples: $x = y$ $y = 4$ $x = -2$ $y = 2x - 7$ $y + x = 10$ $2y - 4x = 12$
3. Quadratic Graph	A '**U-shaped**' curve called a **parabola**. The equation is of the form $y = \boldsymbol{ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0}$**,** the parabola is **upside down**.	
4. Cubic Graph	The equation is of the form $\boldsymbol{y = ax^3 + k}$, where $\boldsymbol{k}$ **is an number**. If $\boldsymbol{a > 0}$, the curve is **increasing**. If $\boldsymbol{a < 0}$, the curve is **decreasing**.	
5. Reciprocal Graph	The equation is of the form $\boldsymbol{y = \frac{A}{x}}$, where $\boldsymbol{A}$ **is a number** and $\boldsymbol{x \neq 0}$. The graph has **asymptotes** on the **x-axis and y-axis**.	
6. Asymptote	A **straight line** that a graph **approaches** but **never touches**.	

7. Exponential Graph	The equation is of the form $\boldsymbol{y = a^x}$, where a is a number called the **base**. If $\boldsymbol{a > 1}$ the graph **increases**. If $\boldsymbol{0 < a < 1}$, the graph **decreases**. The graph has an **asymptote** which is the **x-axis**.	 
8. $y = \sin x$	Key Coordinates: $(\mathbf{0},\mathbf{0}), (\mathbf{90},\mathbf{1}), (\mathbf{180},\mathbf{0}), (\mathbf{270},\mathbf{-1}), (\mathbf{360},\mathbf{0}$ y is never more than 1 or less than -1. Pattern repeats every 360°.	
9. $y = \cos x$	Key Coordinates: $(\mathbf{0},\mathbf{1}), (\mathbf{90},\mathbf{0}), (\mathbf{180},\mathbf{-1}), (\mathbf{270},\mathbf{0}), (\mathbf{360},\mathbf{1}$ y is never more than 1 or less than -1. Pattern repeats every 360°.	
10. $y = \tan x$	Key Coordinates: $(\mathbf{0},\mathbf{0}), (\mathbf{45},\mathbf{1}), (\mathbf{135},\mathbf{-1}), (\mathbf{180},\mathbf{0}),$ $(\mathbf{225},\mathbf{1}), (\mathbf{315},\mathbf{-1}), (\mathbf{360},\mathbf{0})$ **Asymptotes** at $\boldsymbol{x = 90}$ and $\boldsymbol{x = 270}$ Pattern repeats every 360°.	
11. $f(x) + a$	**Vertical translation** up a units. $\begin{pmatrix} 0 \\ a \end{pmatrix}$	
12. $f(x + a)$	**Horizontal translation** <u>left</u> a units. $\begin{pmatrix} -a \\ 0 \end{pmatrix}$	
13. $-f(x)$	**Reflection** over the **x-axis**.	
14. $f(-x)$	**Reflection** over the **y-axis**.	

Topic: Area Under Graph and Gradient of Curve

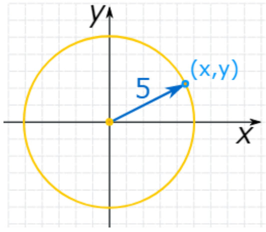
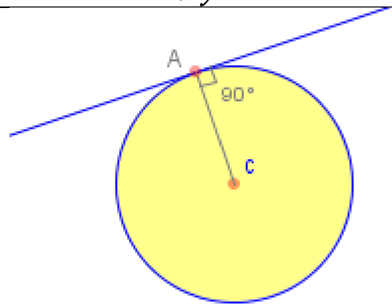
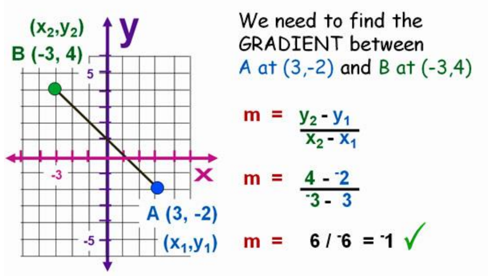
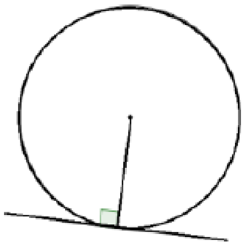
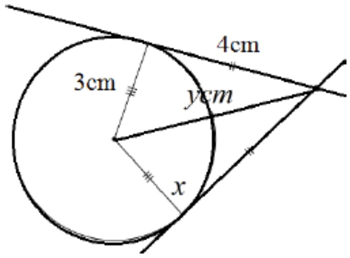
Topic/Skill	Definition/Tips	Example
1. Area Under a Curve	To find the area under a curve, **split it up into simpler shapes** – such as rectangles, triangles and trapeziums – that approximate the area.	
2. Tangent to a Curve	A straight **line** that **touches** a curve at **exactly one point**.	
3. Gradient of a Curve	The **gradient of a curve** at a point is the same as the **gradient of the tangent** at that point. 1. Draw a tangent carefully at the point. 2. Make a right-angled triangle. 3. Use the measurements on the axes to calculate the rise and run (change in y and change in x) 4. Calculate the gradient.	 $Gradient = \frac{Change\ in\ y}{Change\ in\ x}$ $= \frac{16}{2} = 8$

4. Rate of Change	The rate of change at a particular instant in time is represented by the **gradient of the tangent to the curve** at that point.	 
5. Distance-Time Graphs	You can find the **speed** from the **gradient** of the line (Distance ÷ Time) The steeper the line, the quicker the speed. A **horizontal** line means the object is not moving (**stationary**).	
6. Velocity-Time Graphs	You can find the **acceleration** from the **gradient** of the line (Change in Velocity ÷ Time) The steeper the line, the quicker the acceleration. A **horizontal line** represents no acceleration, meaning a **constant velocity**. The **area** under the graph is the **distance.**	

Subject: Maths **Topic: Iteration**

Topic/Skill	Definition/Tips	Example
1. Iteration	The act of **repeating a process** over and over again, often with the aim of **approximating** a desired result more closely. **Recursive** Notation: $x_{n+1} = \sqrt{3x_n + 6}$	$x_1 = 4$ $x_2 = \sqrt{3 \times 4 + 6} = 4.242640 \ldots$ $x_3 = \sqrt{3 \times 4.242640 \ldots + 6}$ $= 4.357576 \ldots$
2. Iterative Method	To create an iterative formula, **rearrange** an equation with more than one x term to **make one of the x terms the subject**. You will be given the first value to substitute in, often called $\boldsymbol{x_1}$. **Keep substituting in your previous answer** until your answers are the same to a certain degree of accuracy. This is called converging to a limit. Use the 'ANS' button on your calculator to keep substituting in the previous answer.	Use an iterative formula to find the positive root of $x^2 - 3x - 6 = 0$ to 3 decimal places. $x_1 = 4$ Answer: $x^2 = 3x + 6$ $x = \sqrt{3x + 6}$ So $x_{n+1} = \sqrt{3x_n + 6}$ $x_1 = 4$ $x_2 = \sqrt{3 \times 4 + 6} = 4.242640 \ldots$ $x_3 = \sqrt{3 \times 4.242640 \ldots + 6}$ $= 4.357576 \ldots$ Keep repeating… $x_7 = 4.372068.. = 4.372\ (3dp)$ $x_8 = 4.372208 \ldots = 4.372\ (3dp)$ So answer is $x = 4.372\ (3dp)$

Subject: Maths **Topic: Equation of a Circle and Tangent**

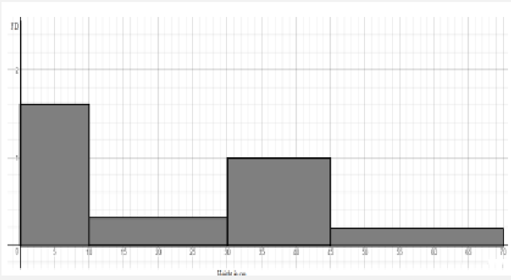
Topic/Skill	Definition/Tips	Example
1. Equation of a Circle	The equation of a **circle**, **centre (0,0), radius r**, is: $$x^2 + y^2 = r^2$$	 $x^2 + y^2 = 25$
2. Tangent	A straight **line** that **touches** a circle at **exactly one point**, never entering the circle's interior. A **radius** is **perpendicular** to a **tangent** at the **point of contact**.	
3. Gradient	**Gradient** is another word for **slope**. $$G = \frac{Rise}{Run} = \frac{Change\ in\ y}{Change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$	
4. Circle Theorem 5	**A tangent is perpendicular to the radius at the point of contact.** 	 $y = 5cm$ (Pythagoras' Theorem)

Topic: Histograms and Cumulative Frequency

Topic/Skill	Definition/Tips	Example
1. Histograms	A visual way to display frequency data using bars. Bars can be **unequal in width**. Histograms show **frequency density** on the **y-axis**, not frequency. $\boldsymbol{Frequency\ Density = \frac{Frequency}{Class\ Width}}$	See tables and histogram below

Height(cm)	Frequency
$0 < h \le 10$	8
$10 < h \le 30$	6
$30 < h \le 45$	15
$45 < h \le 70$	5

Frequency Density (FD)
$8 \div 5 = 1.6$
$6 \div 20 = 0.3$
$15 \div 15 = 1$
$5 \div 25 = 0.2$

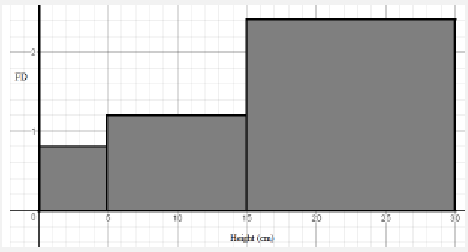


2. Interpreting Histograms

The **area** of the bar is proportional to the **frequency** of that class interval.

$$\boldsymbol{Frequency = Freq\ Density \times Class\ Width}$$

A histogram shows information about the heights of a number of plants. 4 plants were less than 5cm tall. Find the number of plants more than 5cm tall.



Above 5cm:
1.2 x 10 + 2.4 x 15 = 12 + 36 = 48

3. Cumulative Frequency

Cumulative Frequency is a **running total**.

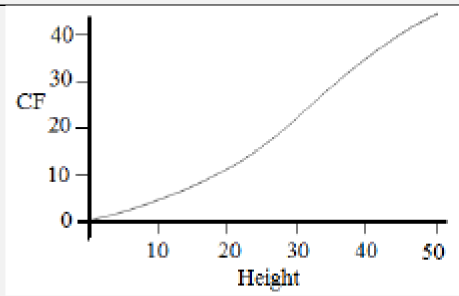
Age	Frequency
$0 < a \le 10$	15
$10 < a \le 40$	35
$40 < a \le 50$	10

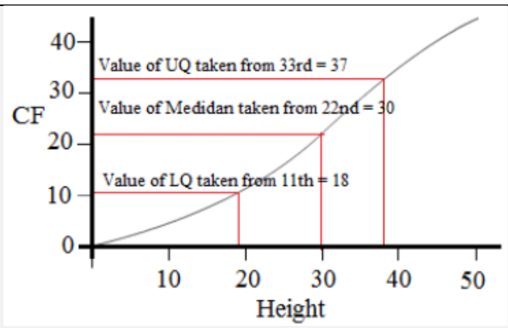
Cumulative Frequency
15
$15 + 35 = 50$
$50 + 10 = 60$

4. Cumulative Frequency Diagram

A cumulative frequency diagram is a **curve that goes up**. It looks a little like a stretched-out **S shape**.

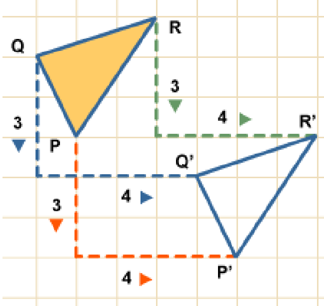
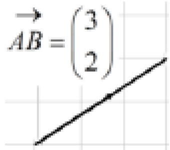
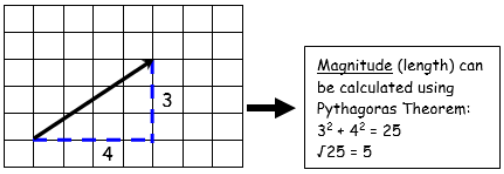
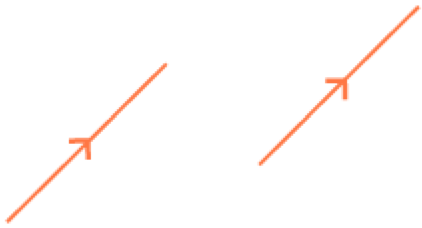
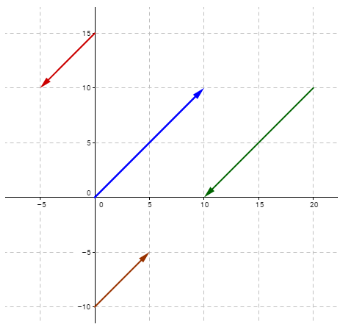
Plot the cumulative frequencies at the **end-point** of each interval.

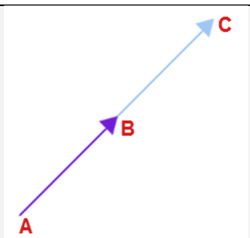
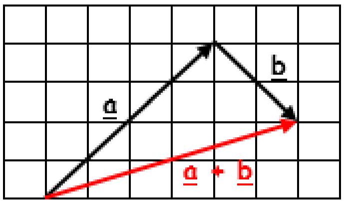
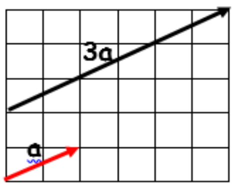
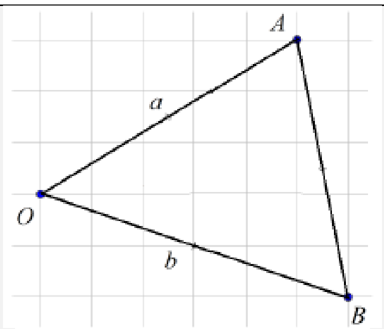
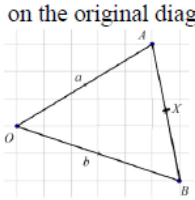


5. Quartiles from Cumulative Frequency Diagram	**Lower Quartile** (Q1): **25%** of the data is less than the lower quartile. **Median** (Q2): **50%** of the data is less than the median. **Upper Quartile** (Q3): **75%** of the data is less than the upper quartile. **Interquartile Range (IQR)**: represents the **middle 50%** of the data.	 $IQR = 37 - 18 = 19$
6. Hypothesis	**A statement that might be true, which can be tested.**	Hypothesis: ‘Large dogs are better at catching tennis balls than small dogs’. We can test this hypothesis by having hundreds of different sized dogs try to catch tennis balls.

Subject: Maths **Topic: Proofs**

Topic/Skill	Definition/Tips	Example
1. Expression	A mathematical statement written using **symbols**, **numbers** or **letters**,	3x + 2 or $5y^2$
2. Equation	A statement showing that **two expressions are equal**	2y – 17 = 15
3. Identity	An equation that is **true for all values** of the variables An identity uses the symbol: ≡	$2x \equiv x+x$
4. Formula	Shows the **relationship** between **two or more variables**	Area of a rectangle = length x width or A= LxW
5. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient z is the variable
6. Odds and Evens	An **even** number is a **multiple of 2** An **odd** number is an integer which is **not a multiple of 2**.	If n is an integer (whole number): An even number can be represented by **2n** or **2m** etc. An odd number can be represented by **2n-1** or **2n+1** or **2m+1** etc.
7. Consecutive Integers	Whole numbers that follow each other in order.	If n is an integer: **n, n+1, n+2** etc. are consecutive integers.
8. Square Terms	A term that is produced by multiply another term by itself.	If n is an integer: n^2, m^2 etc. are square integers
9. Sum	The sum of two or more numbers is the value you get when you add them together.	The sum of 4 and 6 is 10
10. Product	The product of two or more numbers is the value you get when you multiply them together.	The product of 4 and 6 is 24
11. Multiple	To show that an expression is a **multiple** of a number, you need to show that you can **factor out the number**.	$4n^2 + 8n - 12$ is a multiple of 4 because it can be written as: $4(n^2 + 2n - 3)$

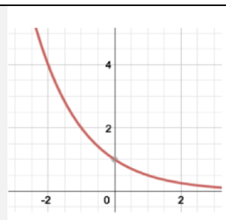
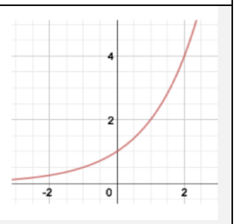
Topic/Skill	**Definition/Tips**	**Example**
1. Translation	**Translate** means to **move a shape**. The shape does not change **size** or **orientation**.	
2. Vector Notation	A vector can be written in 3 ways: **a** or $\overrightarrow{\boldsymbol{AB}}$ or $\begin{pmatrix} \mathbf{1} \\ \mathbf{3} \end{pmatrix}$	
3. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ means ‘2 right, 3 up’ $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$ means ‘1 left, 5 down’
4. Vector	A **vector** is a quantity represented by an arrow with both **direction** and **magnitude**. $\overrightarrow{AB} = -\overrightarrow{BA}$	
5. Magnitude	Magnitude is defined as the **length** of a vector.	
6. Equal Vectors	If two vectors have the **same magnitude and direction**, they are **equal**.	
7. Parallel Vectors	**Parallel** vectors are **multiples** of each other.	2**a**+**b** and 4**a**+2**b** are parallel as they are multiple of each other. 

8. Collinear Vectors	**Collinear** vectors are vectors that are on the **same line**. To show that two vectors are **collinear**, show that one vector is a **multiple** of the other (parallel) **AND** that both vectors **share a point**.	
9. Resultant Vector	The **resultant** vector is the vector that results from **adding** two or more vectors together. The resultant can also be shown by **lining up** the **head** of one vector with the **tail** of the other.	if **a** = $\binom{4}{4}$ and **b** = $\binom{2}{-2}$ then **a** + **b** = $\binom{4}{4} + \binom{2}{-2} = \binom{6}{2}$ 
10. Scalar of a Vector	A **scalar** is the **number** we **multiply** a vector by.	 Example: 3a + 2b = $= 3\binom{2}{1} + 2\binom{4}{-1}$ $= \binom{6}{3} + \binom{8}{-2}$ $= \binom{14}{1}$
11. Vector Geometry	 $\overrightarrow{OA} = a \quad \overrightarrow{AO} = -a$ $\overrightarrow{OB} = b \quad \overrightarrow{BO} = -b$ $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$ $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$	**Example 1:** X is the midpoint of AB. Find $\overrightarrow{OX}$ **Answer:** Draw X on the original diagram  Now build up a journey. You could use $\overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$. This will give: $\overrightarrow{OX} = a + \frac{1}{2}(b-a)$. This will simplify to $\frac{1}{2}a + \frac{1}{2}b$ or $\frac{1}{2}(a+b)$

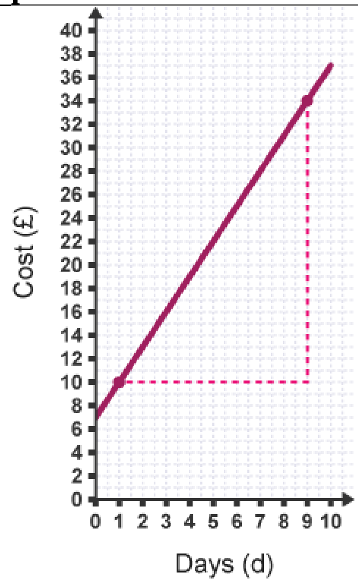
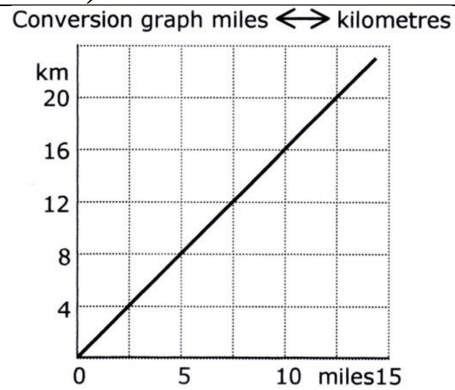
Topic: Algebraic Fractions

Topic/Skill	Definition/Tips	Example
1. Algebraic Fraction	A fraction whose **numerator** and **denominator** are **algebraic expressions**.	$\frac{6x}{3x-1}$
2. Adding/ Subtracting Algebraic Fractions	For $\frac{a}{b} \pm \frac{c}{d}$, the **common denominator** is bd $\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}$	$\frac{1}{x} + \frac{x}{2y}$ $= \frac{1(2y)}{2xy} + \frac{x(x)}{2xy}$ $= \frac{2y + x^2}{2xy}$
3. Multiplying Algebraic Fractions	**Multiply** the **numerators together** and the **denominators together**. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	$\frac{x}{3} \times \frac{x+2}{x-2}$ $= \frac{x(x+2)}{3(x-2)}$ $= \frac{x^2 + 2x}{3x - 6}$
4. Dividing Algebraic Fractions	**Multiply** the first fraction by the **reciprocal of the second fraction**. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$	$\frac{x}{3} \div \frac{2x}{7}$ $= \frac{x}{3} \times \frac{7}{2x}$ $= \frac{7x}{6x} = \frac{7}{6}$
5. Simplifying Algebraic Fractions	**Factorise** the numerator and denominator and **cancel common factors**.	$\frac{x^2 + x - 6}{2x - 4} = \frac{(x+3)(x-2)}{2(x-2)} = \frac{x+3}{2}$

Topic: Growth and Decay

Topic/Skill	Definition/Tips	Example
1. Exponential Growth	When we **multiply** a number **repeatedly** by the **same number** ($\neq 1$), resulting in the number **increasing by the same proportion** each time. The original amount can grow very quickly in exponential growth.	$1, 2, 4, 8, 16, 32, 64, 128$ ... is an example of exponential growth, because the numbers are being multiplied by 2 each time.
2. Exponential Decay	When we **multiply** a number **repeatedly** by the **same number** ($\mathbf{0 < x < 1}$), resulting in the number **decreasing by the same proportion** each time. The original amount can decrease very quickly in exponential decay.	$1000, 200, 40, 8$ ... is an example of exponential decay, because the numbers are being multiplied by $\frac{1}{5}$ each time.
3. Compound Interest	Interest paid on the **original amount and the accumulated interest**.	A bank pays 5% compound interest a year. Bob invests £3000. How much will he have after 7 years. $3000 \times 1.05^7 = £4221.30$
4. Exponential Graph	The equation is of the form $\boldsymbol{y = a^x}$**,** where $\boldsymbol{a}$ is a number called the **base**. If $\boldsymbol{a > 1}$ the graph **increases**. If $\boldsymbol{0 < a < 1}$, the graph **decreases**. The graph has an **asymptote** which is the **x-axis**. The **y-intercept** of the graph $y = a^x$ is $\mathbf{(0, 1)}$s	 

Topic: Real Life Graphs

Topic/Skill	Definition/Tips	Example
1. Real Life Graphs	Graphs that are supposed to model some real-life situation. The actual meaning of the values depends on the labels and units on each axis. The **gradient** might have a contextual meaning. The **y-intercept** might have a contextual meaning. The **area** under the graph might have a contextual meaning.	 A graph showing the cost of hiring a ladder for various numbers of days. The gradient shows the cost per day. It costs £3/day to hire the ladder. The y-intercept shows the additional cost/deposit/fixed charge (something not linked to how long the ladder is hired for). The additional cost is £7.
2. Conversion Graph	A line graph to **convert one unit to another**. Can be used to convert units (eg. miles and kilometres) or currencies ($ and £) Find the value you know on one axis, read up/across to the conversion line and read the equivalent value from the other axis.	 $8\ km = 5\ miles$
3. Depth of Water in Containers	Graphs can be used to show how the depth of water changes as different shaped containers are filled with water at a constant rate.