Mathematics
RAKK

| Topic/Skill | Definition/Tips | Example |  |
| :--- | :--- | :--- | :--- |
| 1. Scale | The ratio of the length in a model to the <br> length of the real thing. | The ratio of a distance on the map to the <br> actual distance in real life. | Real Horse <br> 1500 <br> 2000 mm high |
| 2. Scale (Map) |  |  |  |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Congruent Shapes | Shapes are congruent if they are identical same shape and same size. <br> Shapes can be rotated or reflected but still be congruent. |  |
| 2. Congruent Triangles | 4 ways of proving that two triangles are congruent: <br> 1. SSS (Side, Side, Side) <br> 2. RHS (Right angle, Hypotenuse, Side) <br> 3. SAS (Side, Angle, Side) <br> 4. ASA (Angle, Side, Angle) or AAS <br> ASS does not prove congruency. | $\begin{aligned} & B C=D F \\ & \angle A B C=\angle E D F \\ & \angle A C B=\angle E F D \end{aligned}$ <br> $\therefore$ The two triangles are congruent by AAS. |
| 3. Similar Shapes | Shapes are similar if they are the same shape but different sizes. <br> The proportion of the matching sides must be the same, meaning the ratios of corresponding sides are all equal. |  |
| 4. Scale Factor | The ratio of corresponding sides of two similar shapes. <br> To find a scale factor, divide a length on one shape by the corresponding length on a similar shape. | Scale Factor $=15 \div 10=1.5$ |
| 5. Finding missing lengths in similar shapes | 1. Find the scale factor. <br> 2. Multiply or divide the corresponding side to find a missing length. <br> If you are finding a missing length on the larger shape you will need to multiply by the scale factor. <br> If you are finding a missing length on the smaller shape you will need to divide by the scale factor. | Scale Factor $=3 \div 2=1.5$ $x=4.5 \times 1.5=6.75 \mathrm{~cm}$ |
| 6. Similar Triangles | To show that two triangles are similar, show that: <br> 1. The three sides are in the same proportion <br> 2. Two sides are in the same proportion, and their included angle is the same <br> 3. The three angles are equal |  |


| Topic/Skill | Definition/Tips | Parallel lines never meet. |
| :--- | :--- | :--- |
| . Parallel | Perpendicular lines are at right angles. <br> There is a $90^{\circ}$ angle between them. |  |
| 2. <br> Perpendicular |  |  |
| 3. Vertex | A corner or a point where two lines meet. <br> Angle Bisector: Cuts the angle in half. <br> 3. Place the sharp end of a pair of <br> compasses on the vertex. <br> 2. Draw an arc, marking a point on each <br> line. <br> 3. Without changing the compass put the <br> compass on each point and mark a centre <br> point where two arcs cross over. <br> 4. Use a ruler to draw a line through the <br> vertex and centre point. |  |
| 5. |  |  |
| Perpendicular Bisector: Cuts a line in <br> half and at right angles. <br> Bisector | 1. Put the sharp point of a pair of <br> compasses on A. <br> 2. Open the compass over half way on the <br> line. <br> 3. Draw an arc above and below the line. <br> 4. Without changing the compass, repeat <br> from point B. <br> 5. Draw a straight line through the two <br> intersecting arcs. |  |
| The perpendicular distance from a point <br> to a line is the shortest distance to that <br> line. <br> 1. Put the sharp point of a pair of <br> compasses on the point. <br> 2. Draw an arc that crosses the line twice. <br> 3. Place the sharp point of the compass on <br> one of these points, open over half way and <br> draw an arc above and below the line. <br> 4. Repeat from the other point on the line. |  |  |
| Perpendicula |  |  |
| from an |  |  |
| External Point |  |  |


|  | 5. Draw a straight line through the two <br> intersecting arcs. |
| :--- | :--- |
| 7. <br> Perpendicular <br> from a Point <br> on a Line | 1. Put the sharp point of a pair of <br> compasses on point R. <br> 2. Draw two arcs either side of the point of <br> equal width (giving points S and T) <br> 3. Place the compass on point S, open over <br> halfway and draw an arc above the line. <br> 4. Repeat from the other arc on the line <br> (point T). <br> 5. Draw a straight line from the intersecting <br> arcs to the original point on the line. |
| 8. Constructing <br> Triangles <br> (Side, Side, | 1. Draw the base of the triangle using a <br> ruler. <br> 2. Open a pair of compasses to the width of <br> one side of the triangle. <br> 3. Place the point on one end of the line and <br> draw an arc. <br> 4. Repeat for the other side of the triangle <br> at the other end of the line. |
| 5. Using a ruler, draw lines connecting the |  |
| ends of the base of the triangle to the point |  |
| where the arcs intersect. |  |,


| 11. <br> Constructing <br> an Equilateral <br> Triangle (also <br> makes a $60^{\circ}$ <br> angle) | 1. Draw the base of the triangle using a <br> ruler. <br> 2. Open the pair of compasses to the exact <br> length of the side of the triangle. <br> 3. Place the sharp point on one end of the <br> line and draw an arc. <br> 4. Repeat this from the other end of the <br> line. <br> 5. Using a ruler, draw lines connecting the <br> ends of the base of the triangle to the point <br> where the arcs intersect. |
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| 12. Loci and <br> Regions <br> rule. <br> For the locus of points closer to $\mathbf{B}$ than $\mathbf{A}$, <br> create a perpendicular bisector between A <br> and B and shade the side closer to B. |  |
| For the locus of points equidistant from $\mathbf{A}$, |  |
| use a compass to draw a circle, centre A. |  |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Translation | Translate means to move a shape. The shape does not change size or orientation. |  |
| 2. Column Vector | In a column vector, the top number moves left (-) or right (+) and the bottom number moves up (+) or down (-) | $\binom{2}{3}$ means ' 2 right, 3 up' $\binom{-1}{-5}$ means ' 1 left, 5 down' |
| 3. Rotation | The size does not change, but the shape is turned around a point. <br> Use tracing paper. | Rotate Shape A $90^{\circ}$ anti-clockwise about $(0,1)$ |
| 4. Reflection | The size does not change, but the shape is 'flipped' like in a mirror. <br> Line $\boldsymbol{x}=$ ? is a vertical line. <br> Line $\boldsymbol{y}=$ ? is a horizontal line. <br> Line $\boldsymbol{y}=\boldsymbol{x}$ is a diagonal line. | Reflect shape C in the line $y=x$ |
| 5. Enlargement | The shape will get bigger or smaller. Multiply each side by the scale factor. | ```Scale Factor = 3 means ' }3\mathrm{ times larger = multiply by 3' Scale Factor = 1/2 means 'half the size = divide by 2'``` |


| 6. Finding the Centre of Enlargement | Draw straight lines through corresponding corners of the two shapes. The centre of enlargement is the point where all the lines cross over. <br> Be careful with negative enlargements as the corresponding corners will be the other way around. |  |
| :---: | :---: | :---: |
| 7. Describing Transformatio ns | Give the following information when describing each transformation: <br> Look at the number of marks in the question for a hint of how many pieces of information are needed. <br> If you are asked to describe a 'transformation', you need to say the name of the type of transformation as well as the other details. | - Translation, Vector <br> - Rotation, Direction, Angle, Centre <br> - Reflection, Equation of mirror line <br> - Enlargement, Scale factor, Centre of enlargement |
| 8. Negative Scale Factor Enlargements | Negative enlargements will look like they have been rotated. <br> $S F=-2$ will be rotated, and also twice as big. | Enlarge ABC by scale factor -2 , centre <br> $(1,1)$ |
| 9. Invariance | A point, line or shape is invariant if it does not change/move when a transformation is performed. <br> An invariant point 'does not vary'. | If shape P is reflected in the $y-$ axis, then exactly one vertex is invariant. |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. <br> Trigonometry | The study of triangles. |  |
| 2. Hypotenuse | The longest side of a right-angled triangle. <br> Is always opposite the right angle. |  |
| 3. Adjacent | Next to |  |
| 4. <br> Trigonometric Formulae | Use SOHCAHTOA. $\begin{aligned} & \sin \theta=\frac{O}{H} \\ & \cos \theta=\frac{A}{H} \\ & \tan \theta=\frac{O}{A} \end{aligned}$ <br> When finding a missing angle, use the 'inverse' trigonometric function by pressing the 'shift' button on the calculator. | Use 'Opposite' and 'Adjacent', so use 'tan' $\begin{gathered} \tan 35=\frac{x}{11} \\ x=11 \tan 35=7.70 \mathrm{~cm} \end{gathered}$ use 'cos' $\begin{gathered} \cos x=\frac{5}{7} \\ x=\cos ^{-1}\left(\frac{5}{7}\right)=44.4^{\circ} \end{gathered}$ <br> Use 'Adjacent' and 'Hypotenuse', so |
| $\begin{aligned} & \text { 5. 3D } \\ & \text { Trigonometry } \end{aligned}$ | Find missing lengths by identifying right angled triangles. <br> You will often have to find a missing length you are not asked for before finding the missing length you are asked for. |  |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Quadratic | A quadratic expression is of the form $a x^{2}+b x+c$ <br> where $a, b$ and $c$ are numbers, $\boldsymbol{a} \neq \mathbf{0}$ | Examples of quadratic expressions: $\begin{gathered} x^{2} \\ 8 x^{2}-3 x+7 \end{gathered}$ <br> Examples of non-quadratic expressions: $\begin{gathered} 2 x^{3}-5 x^{2} \\ 9 x-1 \\ \hline \end{gathered}$ |
| 2. Factorising Quadratics | When a quadratic expression is in the form $x^{2}+b x+c$ find the two numbers that add to give $\mathbf{b}$ and multiply to give $\mathbf{c}$. | $x^{2}+7 x+10=(x+5)(x+2)$ <br> (because 5 and 2 add to give 7 and multiply to give 10 ) $x^{2}+2 x-8=(x+4)(x-2)$ <br> (because +4 and -2 add to give +2 and multiply to give -8) |
| 3. Difference of Two Squares | An expression of the form $\boldsymbol{a}^{2}-\boldsymbol{b}^{2}$ can be factorised to give $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{a}-\boldsymbol{b})$ | $\begin{aligned} x^{2}-25 & =(x+5)(x-5) \\ 16 x^{2}-81 & =(4 x+9)(4 x-9) \end{aligned}$ |
| 4. Solving Quadratics ( $a x^{2}=b$ ) | Isolate the $x^{2}$ term and square root both sides. <br> Remember there will be a positive and a negative solution. | $\begin{gathered} 2 x^{2}=98 \\ x^{2}=49 \\ x= \pm 7 \end{gathered}$ |
| 5. Solving Quadratics $\left(a x^{2}+b x=\right.$ 0 ) | Factorise and then solve $=0$. | $\begin{gathered} x^{2}-3 x=0 \\ x(x-3)=0 \\ x=0 \text { or } x=3 \end{gathered}$ |
| 6. Solving Quadratics by Factorising ( $a=1$ ) | Factorise the quadratic in the usual way. Solve $=0$ <br> Make sure the equation $=0$ before factorising. | Solve $x^{2}+3 x-10=0$ <br> Factorise: $\begin{gathered} (x+5)(x-2)=0 \\ x=-5 \text { or } x=2 \end{gathered}$ |
| 7. Quadratic Graph | A 'U-shaped' curve called a parabola. <br> The equation is of the form $y=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c}$, where $a, b$ and $c$ are numbers, $\boldsymbol{a} \neq \mathbf{0}$. <br> If $\boldsymbol{a}<\mathbf{0}$, the parabola is upside down. |  |
| 8. Roots of a Quadratic | A root is a solution. <br> The roots of a quadratic are the $\boldsymbol{x}$ intercepts of the quadratic graph. |  |


| 9. Turning Point of a Quadratic | A turning point is the point where a quadratic turns. <br> On a positive parabola, the turning point is called a minimum. <br> On a negative parabola, the turning point is called a maximum. |  |
| :---: | :---: | :---: |
| 10. Factorising Quadratics when $a \neq 1$ | When a quadratic is in the form $a x^{2}+b x+c$ <br> 1. Multiply a by $\mathrm{c}=\mathrm{ac}$ <br> 2. Find two numbers that add to give $b$ and multiply to give ac. <br> 3. Re-write the quadratic, replacing $b x$ with the two numbers you found. <br> 4. Factorise in pairs - you should get the same bracket twice <br> 5. Write your two brackets - one will be the repeated bracket, the other will be made of the factors outside each of the two brackets. | $\text { Factorise } 6 x^{2}+5 x-4$ <br> 1. $6 \times-4=-24$ <br> 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 <br> 3. $6 x^{2}+8 x-3 x-4$ <br> 4. Factorise in pairs: $\begin{gathered} 2 x(3 x+4)-1(3 x+4) \\ \text { 5. Answer }=(3 x+4)(2 x-1) \end{gathered}$ |
| 11. Solving Quadratics by Factorising $(a \neq 1)$ | Factorise the quadratic in the usual way. Solve $=0$ <br> Make sure the equation $=0$ before factorising. | Solve $2 x^{2}+7 x-4=0$ <br> Factorise: $\begin{aligned} & (2 x-1)(x+4)=0 \\ & x=\frac{1}{2} \text { or } x=-4 \end{aligned}$ |
| 12. Completing the Square (when $a=1$ ) | A quadratic in the form $x^{2}+b x+c$ can be written in the form $(\boldsymbol{x}+\boldsymbol{p})^{2}+\boldsymbol{q}$ <br> 1. Write a set of brackets with $x$ in and half the value of $b$. <br> 2. Square the bracket. <br> 3. Subtract $\left(\frac{b}{2}\right)^{2}$ and add $c$. <br> 4. Simplify the expression. <br> You can use the completing the square form to help find the maximum or minimum of quadratic graph. | Complete the square of $y=x^{2}-6 x+2$ <br> Answer: $\begin{aligned} & (x-3)^{2}-3^{2}+2 \\ & =(x-3)^{2}-7 \end{aligned}$ <br> The minimum value of this expression occurs when $(x-3)^{2}=0$, which occurs when $x=3$ <br> When $x=3, y=0-7=-7$ $\text { Minimum point }=(3,-7)$ |
| 13. Completing the Square (when $a \neq 1$ ) | A quadratic in the form $a x^{2}+b x+c$ can be written in the form $\mathbf{p}(\boldsymbol{x}+\boldsymbol{q})^{2}+\boldsymbol{r}$ <br> Use the same method as above, but factorise out $a$ at the start. | Complete the square of $4 x^{2}+8 x-3$ <br> Answer: $\begin{aligned} & 4\left[x^{2}+2 x\right]-3 \\ = & 4\left[(x+1)^{2}-1^{2}\right]-3 \\ = & 4(x+1)^{2}-4-3 \\ = & 4(x+1)^{2}-7 \end{aligned}$ |
| 14. Solving Quadratics by Completing the Square | Complete the square in the usual way and use inverse operations to solve. | Solve $x^{2}+8 x+1=0$ <br> Answer: $\begin{gathered} (x+4)^{2}-4^{2}+1=0 \\ (x+4)^{2}-15=0 \end{gathered}$ |


|  |  | $\begin{gathered} (x+4)^{2}=15 \\ (x+4)= \pm \sqrt{15} \\ x=-4 \pm \sqrt{15} \end{gathered}$ |
| :---: | :---: | :---: |
| 15. Solving Quadratics using the Quadratic Formula | A quadratic in the form $a x^{2}+b x+c=0$ can be solved using the formula: $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ <br> Use the formula if the quadratic does not factorise easily. | Solve $3 x^{2}+x-5=0$ <br> Answer: $\begin{aligned} & a=3, b=1, c=-5 \\ & x=\frac{-1 \pm \sqrt{1^{2}-4 \times 3 \times-5}}{2 \times 3} \\ & x=\frac{-1 \pm \sqrt{61}}{6} \\ & x=1.14 \text { or }-1.47 \text { (2 d.p.) } \end{aligned}$ |


| Topic/Skill | Definition/Tips | Example |
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| Circle <br> Theorem 1 <br> at the circumference. |  |  |
| Circle |  |  |
| Theorem 2 |  |  |


| Circle <br> Theorem 6 | Tangents from an external point at equal <br> in length. |
| :--- | :--- |
| Circle <br> Theorem 7 | Alternate Segment Theorem |

Topic: Inequalities

| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Inequality | An inequality says that two values are not equal. <br> $a \neq b$ means that a is not equal to b . | $\begin{aligned} & 7 \neq 3 \\ & x \neq 0 \end{aligned}$ |
| 2. Inequality symbols | $x>2$ means x is greater than 2 <br> $x<3$ means $x$ is less than 3 <br> $x \geq 1$ means $\mathbf{x}$ is greater than or equal to 1 <br> $x \leq 6$ means $x$ is less than or equal to 6 | State the integers that satisfy $\begin{aligned} & -2<x \leq 4 \\ & -1,0,1,2,3,4 \end{aligned}$ |
| 3. Inequalities on a Number Line | Inequalities can be shown on a number line. <br> Open circles are used for numbers that are less than or greater than $(<o r>)$ <br> Closed circles are used for numbers that are less than or equal or greater than or equal ( $\leq$ or $\geq$ ) |  |
| 4. Graphical Inequalities | Inequalities can be represented on a coordinate grid. <br> If the inequality is strict $(x>2)$ then use a dotted line. <br> If the inequality is not strict $(x \leq 6)$ then use a solid line. <br> Shade the region which satisfies all the inequalities. | Shade the region that satisfies: $y>2 x, x>1 \text { and } y \leq 3$  |
| 5. Quadratic Inequalities | Sketch the quadratic graph of the inequality. <br> If the expression is $>\boldsymbol{o r} \geq$ then the answer will be above the x -axis. <br> If the expression is $<\boldsymbol{o r} \leq$ then the answer will be below the $\mathbf{x}$-axis. <br> Look carefully at the inequality symbol in the question. <br> Look carefully if the quadratic is a positive or negative parabola. | Solve the inequality $x^{2}-x-12<0$ <br> Sketch the quadratic: <br> The required region is below the x -axis, so the final answer is: $-3<x<4$ <br> If the question had been $>0$, the answer would have been: $x<-3 \text { or } x>4$ |
| 6. Set Notation | A set is a collection of things, usually numbers, denoted with brackets $\{\quad\}$ | $\{3,6,9\}$ is a set. |

$\{x \mid x \geq 7\}$ means 'the set of all x 's, such that x is greater than or equal to 7 '

The ' $x$ ' can be replaced by any letter.
Some people use ':' instead of '|'
$\{x:-2 \leq x<5\}$



| 5. Area of a <br> Triangle | Use when given the length of two sides <br> and the included angle. <br> Area of $\boldsymbol{a}$ Triangle $=\frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{a b} \sin \boldsymbol{C}$ |
| :--- | :--- |
| $A=\frac{1}{2} a b \sin C$ |  |
| $A=\frac{1}{2} \times 7 \times 10 \times \sin 25$ |  |
| $A=14.8$ |  |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Tree Diagrams | Tree diagrams show all the possible outcomes of an event and calculate their probabilities. <br> All branches must add up to 1 when adding downwards. <br> This is because the probability of something not happening is $\mathbf{1}$ minus the probability that it does happen. <br> Multiply going across a tree diagram. <br> Add going down a tree diagram. |  |
| 2. Independent Events | The outcome of a previous event does not influence/affect the outcome of a second event. | An example of independent events could be replacing a counter in a bag after picking it. |
| 3. Dependent Events | The outcome of a previous event does influence/affect the outcome of a second event. | An example of dependent events could be not replacing a counter in a bag after picking it. <br> 'Without replacement' |
| 4. Probability Notation | $\mathbf{P}(\mathbf{A})$ refers to the probability that event $\mathbf{A}$ will occur. <br> $\mathbf{P}\left(\mathbf{A}^{\prime}\right)$ refers to the probability that event A will not occur. <br> $\mathbf{P}(\mathbf{A} \cup \mathbf{B})$ refers to the probability that event A or B or both will occur. <br> $\mathbf{P}(\mathbf{A} \cap \mathbf{B})$ refers to the probability that both events $A$ and $B$ will occur. | P (Red Queen) refers to the probability of picking a Red Queen from a pack of cards. <br> P (Blue') refers to the probability that you do not pick Blue. <br> P(Blonde U Right Handed) refers to the probability that you pick someone who is Blonde or Right Handed or both. <br> P(Blonde $\cap$ Right Handed) refers to the probability that you pick someone who is both Blonde and Right Handed. |
| 5. Venn <br> Diagrams | A Venn Diagram shows the relationship between a group of different things and how they overlap. <br> You may be asked to shade Venn Diagrams as shown below and to the right. <br> The Union <br> The Intersection |  <br> $(A \cap B)^{\prime}$ <br> $(A \cup B)^{\prime}$ |


|  |  | $A \cup B^{\prime}$ |
| :---: | :---: | :---: |
| 6. Venn <br> Diagram <br> Notation | $\in$ means 'element of a set' (a value in the set) <br> \{ \} means the collection of values in the set. <br> $\xi$ means the 'universal set' (all the values to consider in the question) <br> A' means 'not in set $A^{\prime}$ ' (called complement) <br> A U B means 'A or B or both' (called Union) <br> $A \cap B$ means ' $A$ and $B$ (called Intersection) | Set A is the even numbers less than 10 . $\mathrm{A}=\{2,4,6,8\}$ <br> Set $B$ is the prime numbers less than 10. $B=\{2,3,5,7\}$ <br> $A \cup B=\{2,3,4,5,6,7,8\}$ <br> $A \cap B=\{2\}$ |
| 7. AND rule for Probability | When two events, A and B, are independent: $P(A \text { and } B)=P(A) \times P(B)$ | What is the probability of rolling a 4 and flipping a Tails? $\begin{gathered} P(4 \text { and Tails })=P(4) \times P(\text { Tails }) \\ =\frac{1}{6} \times \frac{1}{2}=\frac{1}{12} \end{gathered}$ |
| 8. OR rule for Probability | When two events, A and B, are mutually exclusive: $P(A \text { or } B)=P(A)+P(B)$ | What is the probability of rolling a 2 or rolling a 5? $\begin{gathered} P(2 \text { or } 5)=P(2)+P(5) \\ =\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3} \end{gathered}$ |
| 9. Conditional Probability | The probability of an event A happening, given that event B has already happened. <br> With conditional probability, check if the numbers on the second branches of a tree diagram changes. For example, if you have 4 red beads in a bag of 9 beads and pick a red bead on the first pick, then there will be 3 red beads left out of 8 beads on the second pick. |  |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Function Machine | Takes an input value, performs some operations and produces an output value. | INPUT OUTPUT |
| 2. Function | A relationship between two sets of values. | $f(x)=3 x^{2}-5$ <br> 'For any input value, square the term, then multiply by 3 , then subtract $5^{\prime}$. |
| 3. Function notation | $f(x)$ <br> $\boldsymbol{x}$ is the input value $\boldsymbol{f}(\boldsymbol{x})$ is the output value. | $f(x)=3 x+11$ <br> Suppose the input value is $x=5$ <br> The output value is $f(5)=3 \times 5+$ $11=26$ |
| 4. Inverse function | $f^{-1}(x)$ <br> A function that performs the opposite process of the original function. <br> 1. Write the function as $y=f(x)$ <br> 2. Rearrange to make $x$ the subject. <br> 3. Replace the $\boldsymbol{y}$ with $\boldsymbol{x}$ and the $\boldsymbol{x}$ with $f^{-1}(x)$ | $f(x)=(1-2 x)^{5}$. Find the inverse. $\begin{aligned} & y=(1-2 x)^{5} \\ & \sqrt[5]{y}=1-2 x \\ & 1-\sqrt[5]{y}=2 x \\ & \frac{1-\sqrt[5]{y}}{2}=x \end{aligned}$ $f^{-1}(x)=\frac{1-\sqrt[5]{x}}{2}$ |
| 5. Composite function | A combination of two or more functions to create a new function. <br> $\boldsymbol{f} \boldsymbol{g}(\boldsymbol{x})$ is the composite function that substitutes the function $\boldsymbol{g}(\boldsymbol{x})$ into the function $f(x)$. <br> $\boldsymbol{f} \boldsymbol{g}(\boldsymbol{x})$ means 'do g first, then f ' $\boldsymbol{g} \boldsymbol{f}(\boldsymbol{x})$ means 'do f first, then g ' | $f(x)=5 x-3, g(x)=\frac{1}{2} x+1$ <br> What is $f g(4)$ ? $\begin{gathered} g(4)=\frac{1}{2} \times 4+1=3 \\ f(3)=5 \times 3-3=12=f g(4) \end{gathered}$ <br> What is $f g(x)$ ? $f g(x)=5\left(\frac{1}{2} x+1\right)-3=\frac{5}{2} x+2$ |


| Topic/Skill | Definition/Tips | Example |
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| 1. Coordinates | Written in pairs. The first term is the $\mathbf{x}$ coordinate (movement across). The second term is the y-coordinate (movement up or down) |  <br> A: $(4,7)$ <br> B: $(-6,-3)$ |
| 2. Linear Graph | Straight line graph. <br> The equation of a linear graph can contain an $\mathbf{x}$-term, a y-term and a number. | Example: <br> Other examples: $\begin{aligned} & x=y \\ & y=4 \\ & x=-2 \\ & y=2 x-7 \\ & y+x=10 \\ & 2 y-4 x=12 \end{aligned}$ |
| 3. Quadratic Graph | A 'U-shaped' curve called a parabola. <br> The equation is of the form $y=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c}$, where $a, b$ and $c$ are numbers, $\boldsymbol{a} \neq \mathbf{0}$. <br> If $\boldsymbol{a}<\mathbf{0}$, the parabola is upside down. |  |
| 4. Cubic Graph | The equation is of the form $\boldsymbol{y}=\boldsymbol{a} x^{3}+\boldsymbol{k}$, where $\boldsymbol{k}$ is an number. <br> If $\boldsymbol{a}>\mathbf{0}$, the curve is increasing. <br> If $\boldsymbol{a}<\mathbf{0}$, the curve is decreasing. |  |
| 5. Reciprocal Graph | The equation is of the form $\boldsymbol{y}=\frac{A}{x}$, where $\boldsymbol{A}$ is a number and $\boldsymbol{x} \neq \mathbf{0}$. <br> The graph has asymptotes on the $\mathbf{x}$-axis and $\mathbf{y}$-axis. |  |
| 6. Asymptote | A straight line that a graph approaches but never touches. |  |


| 7. Exponential Graph | The equation is of the form $\boldsymbol{y}=\boldsymbol{a}^{\boldsymbol{x}}$, where $a$ is a number called the base. <br> If $\boldsymbol{a}>\mathbf{1}$ the graph increases. <br> If $\mathbf{0}<\boldsymbol{a}<\mathbf{1}$, the graph decreases. <br> The graph has an asymptote which is the x-axis. |  |
| :---: | :---: | :---: |
| 8. $y=\sin x$ | ```Key Coordinates: \((0,0),(90,1),(180,0),(270,-1),(360,0\) \(y\) is never more than 1 or less than -1 . Pattern repeats every \(360^{\circ}\).``` |  |
| 9. $y=\cos x$ | Key Coordinates: $(0,1),(90,0),(180,-1),(270,0),(360,1$ <br> $y$ is never more than 1 or less than -1 . <br> Pattern repeats every $360^{\circ}$. |  |
| 10. $y=\tan x$ | $\begin{aligned} & \text { Key Coordinates: } \\ & \quad(\mathbf{0}, \mathbf{0}),(\mathbf{4 5}, \mathbf{1}),(\mathbf{1 3 5},-\mathbf{1}),(\mathbf{1 8 0}, \mathbf{0}) \text {, } \\ & \quad(\mathbf{2 2 5}, \mathbf{1}),(\mathbf{3 1 5},-\mathbf{1}),(\mathbf{3 6 0} \mathbf{0}) \\ & \text { Asymptotes at } \boldsymbol{x}=\mathbf{9 0} \text { and } \boldsymbol{x}=\mathbf{2 7 0} \\ & \text { Pattern repeats every } 360^{\circ} \text {. } \\ & \hline \end{aligned}$ |  |
| 11. $f(x)+a$ | Vertical translation up a units. $\binom{0}{a}$ |  |
| 12. $f(x+a)$ | Horizontal translation left a units. $\binom{-a}{0}$ |  |
| 13. $-f(x)$ | Reflection over the $\mathbf{x}$-axis. |  |
| 14. $f(-x)$ | Reflection over the $\mathbf{y}$-axis. |  |



| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Area Under <br> a Curve | To find the area under a curve, split it up into simpler shapes - such as rectangles, triangles and trapeziums - that approximate the area. |  |
| 2. Tangent to a Curve | A straight line that touches a curve at exactly one point. |  |
| 3. Gradient of a Curve | The gradient of a curve at a point is the same as the gradient of the tangent at that point. <br> 1. Draw a tangent carefully at the point. <br> 2. Make a right-angled triangle. <br> 3. Use the measurements on the axes to calculate the rise and run (change in $y$ and change in $x$ ) <br> 4. Calculate the gradient. |  $\begin{gathered} \text { Gradient }=\frac{\text { Change in } y}{\text { Change in } x} \\ =\frac{16}{2}=8 \end{gathered}$ |



| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Iteration | The act of repeating a process over and over again, often with the aim of approximating a desired result more closely. <br> Recursive Notation: $x_{n+1}=\sqrt{3 x_{n}+6}$ | $\begin{array}{r} x_{1}=4 \\ x_{2}=\sqrt{3 \times 4+6}=4.242640 \ldots \\ x_{3}=\sqrt{3 \times 4.242640 \ldots+6} \\ =4.357576 \ldots \end{array}$ |
| 2. Iterative Method | To create an iterative formula, rearrange an equation with more than one $x$ term to make one of the $x$ terms the subject. <br> You will be given the first value to substitute in, often called $\boldsymbol{x}_{\boldsymbol{1}}$. <br> Keep substituting in your previous answer until your answers are the same to a certain degree of accuracy. This is called converging to a limit. <br> Use the 'ANS' button on your calculator to keep substituting in the previous answer. | Use an iterative formula to find the positive root of $x^{2}-3 x-6=0$ to 3 decimal places. $x_{1}=4$ <br> Answer: $\begin{aligned} & x^{2}=3 x+6 \\ & x=\sqrt{3 x+6} \end{aligned}$ <br> So $x_{n+1}=\sqrt{3 x_{n}+6}$ $\begin{array}{r} x_{1}=4 \\ x_{2}=\sqrt{3 \times 4+6}=4.242640 \ldots \\ x_{3}=\sqrt{3 \times 4.242640 \ldots+6} \\ \quad=4.357576 \ldots \end{array}$ <br> Keep repeating... $\begin{gathered} x_{7}=4.372068 . .=4.372(3 d p) \\ x_{8}=4.372208 \ldots=4.372(3 d p) \end{gathered}$ <br> So answer is $x=4.372(3 d p)$ |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Equation of a Circle | The equation of a circle, centre (0,0), radius $\mathbf{r}$, is: $x^{2}+y^{2}=r^{2}$ |  $x^{2}+y^{2}=25$ |
| 2. Tangent | A straight line that touches a circle at exactly one point, never entering the circle's interior. <br> A radius is perpendicular to a tangent at the point of contact. |  |
| 3. Gradient | Gradient is another word for slope. $G=\frac{\text { Rise }}{\text { Run }}=\frac{\text { Change in } y}{\text { Change in } x}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ |  <br> We need to find the GRADIENT between $A$ at $(3,-2)$ and $B$ at $(-3,4)$ $\begin{aligned} & m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\ & m=\frac{4-2}{3-3} \\ & m=6 / 6=1 \end{aligned}$ |
| 4. Circle Theorem 5 | A tangent is perpendicular to the radius at the point of contact. |  |



| 5. Quartiles from <br> Cumulative <br> Frequency <br> Diagram | Lower Quartile (Q1): 25\% of the data is less than the lower quartile. <br> Median (Q2): $\mathbf{5 0 \%}$ of the data is less than the median. <br> Upper Quartile (Q3): 75\% of the data is less than the upper quartile. <br> Interquartile Range (IQR): represents the middle $50 \%$ of the data. |  $I Q R=37-18=19$ |
| :---: | :---: | :---: |
| 6. Hypothesis | A statement that might be true, which can be tested. | Hypothesis: 'Large dogs are better at catching tennis balls than small dogs'. <br> We can test this hypothesis by having hundreds of different sized dogs try to catch tennis balls. |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Expression | A mathematical statement written using symbols, numbers or letters, | $3 \mathrm{x}+2$ or $5 \mathrm{y}^{2}$ |
| 2. Equation | A statement showing that two expressions are equal | $2 \mathrm{y}-17=15$ |
| 3. Identity | An equation that is true for all values of the variables <br> An identity uses the symbol: $\equiv$ | $2 x \equiv x+x$ |
| 4. Formula | Shows the relationship between two or more variables | Area of a rectangle $=$ length x width or $\mathrm{A}=\mathrm{LxW}$ |
| 5. Coefficient | A number used to multiply a variable. <br> It is the number that comes before/in front of a letter. | $6 z$ <br> 6 is the coefficient z is the variable |
| 6. Odds and Evens | An even number is a multiple of 2 An odd number is an integer which is not a multiple of 2 . | If n is an integer (whole number): <br> An even number can be represented by $\mathbf{2 n}$ or $\mathbf{2 m}$ etc. <br> An odd number can be represented by $\mathbf{2 n - 1}$ or $\mathbf{2 n + 1}$ or $\mathbf{2 m + 1}$ etc. |
| 7. Consecutive Integers | Whole numbers that follow each other in order. | If n is an integer: <br> $\mathbf{n}, \mathbf{n + 1}, \mathbf{n + 2}$ etc. are consecutive integers. |
| 8. Square Terms | A term that is produced by multiply another term by itself. | If n is an integer: <br> $n^{2}, m^{2}$ etc. are square integers |
| 9. Sum | The sum of two or more numbers is the value you get when you add them together. | The sum of 4 and 6 is 10 |
| 10. Product | The product of two or more numbers is the value you get when you multiply them together. | The product of 4 and 6 is 24 |
| 11. Multiple | To show that an expression is a multiple of a number, you need to show that you can factor out the number. | $4 n^{2}+8 n-12$ is a multiple of 4 because it can be written as: $4\left(n^{2}+2 n-3\right)$ |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Translation | Translate means to move a shape. The shape does not change size or orientation. |  |
| 2. Vector Notation | A vector can be written in 3 ways: $\begin{array}{llll} a & \text { or } & \overrightarrow{A B} & \text { or } \end{array}\binom{\mathbf{1}}{\mathbf{3}}$ |  |
| 3. Column Vector | In a column vector, the top number moves left (-) or right (+) and the bottom number moves up (+) or down (-) | $\binom{2}{3}$ means ' 2 right, 3 up' $\binom{-1}{-5}$ means ' 1 left, 5 down' |
| 4. Vector | A vector is a quantity represented by an arrow with both direction and magnitude. $\overrightarrow{A B}=-\overrightarrow{B A}$ | $\overrightarrow{A B}=\binom{3}{2}$ |
| 5. Magnitude | Magnitude is defined as the length of a vector. |  |
| 6. Equal Vectors | If two vectors have the same magnitude and direction, they are equal. |  |
| 7. Parallel Vectors | Parallel vectors are multiples of each other. | $2 \mathbf{a}+\mathbf{b}$ and $4 \mathbf{a}+2 \mathbf{b}$ are parallel as they are multiple of each other. |


| 8. Collinear Vectors | Collinear vectors are vectors that are on the same line. <br> To show that two vectors are collinear, show that one vector is a multiple of the other (parallel) AND that both vectors share a point. |  |
| :---: | :---: | :---: |
| 9. Resultant Vector | The resultant vector is the vector that results from adding two or more vectors together. <br> The resultant can also be shown by lining up the head of one vector with the tail of the other. | if $\underline{a}=\binom{4}{4}$ and $\underline{b}=\binom{2}{-2}$ <br> then $\underline{a}+\underline{b}=\binom{4}{4}+\binom{2}{-2}=\binom{6}{2}$ |
| 10. Scalar of a Vector | A scalar is the number we multiply a vector by. | Example: $\begin{aligned} & 3 a+2 b= \\ & =3\binom{2}{1}+2\binom{4}{-1} \\ & =\binom{6}{3}+\binom{8}{-2} \\ & =\binom{14}{1} \end{aligned}$ |
| 11. Vector Geometry | $\begin{array}{\|ll\|} \hline \overrightarrow{O A}=a & \overrightarrow{A O}=-a \\ \hline \overrightarrow{O B}=b & \overrightarrow{B O}=-b \\ \hline \overrightarrow{A B}=\overrightarrow{A O}+\overrightarrow{O B}=-a+b=b-a \\ \overrightarrow{B A}=\overrightarrow{B O}+\overrightarrow{O A}=-b+a=a-b \\ \hline \end{array}$ | Example 1: $X$ is the midpoint of $A B$. Find $\overrightarrow{O X}$ Answer: Draw $X$ on the original diagram <br> Now build up a journey. <br> You could use $\overrightarrow{O X}=\overrightarrow{O A}+\frac{1}{2} \overrightarrow{A B}$. <br> This will give: $\overrightarrow{O X}=a+\frac{1}{2}(b-a)$. <br> This will simplify to $\frac{1}{2} a+\frac{1}{2} b$ or $\frac{1}{2}(a+b)$ |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Algebraic Fraction | A fraction whose numerator and denominator are algebraic expressions. | $\frac{6 x}{3 x-1}$ |
| 2. Adding/ Subtracting Algebraic Fractions | For $\frac{a}{b} \pm \frac{c}{d}$, the common denominator is bd $\frac{a}{b} \pm \frac{c}{d}=\frac{a d}{b d} \pm \frac{b c}{b d}=\frac{a d \pm b c}{b d}$ | $\begin{gathered} \frac{1}{x}+\frac{x}{2 y} \\ =\frac{1(2 y)}{2 x y}+\frac{x(x)}{2 x y} \\ =\frac{2 y+x^{2}}{2 x y} \end{gathered}$ |
| 3. Multiplying <br> Algebraic <br> Fractions | Multiply the numerators together and the denominators together. $\frac{a}{b} \times \frac{c}{d}=\frac{a c}{b d}$ | $\begin{aligned} & \frac{x}{3} \times \frac{x+2}{x-2} \\ = & \frac{x(x+2)}{3(x-2)} \\ = & \frac{x^{2}+2 x}{3 x-6} \end{aligned}$ |
| 4. Dividing <br> Algebraic <br> Fractions | Multiply the first fraction by the reciprocal of the second fraction. $\frac{a}{b} \div \frac{c}{d}=\frac{a}{b} \times \frac{d}{c}=\frac{a d}{b c}$ | $\begin{aligned} & \frac{x}{3} \div \frac{2 x}{7} \\ = & \frac{x}{3} \times \frac{7}{2 x} \\ = & \frac{7 x}{6 x}=\frac{7}{6} \end{aligned}$ |
| 5. Simplifying Algebraic Fractions | Factorise the numerator and denominator and cancel common factors. | $\frac{x^{2}+x-6}{2 x-4}=\frac{(x+3)(x-2)}{2(x-2)}=\frac{x+3}{2}$ |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Exponential Growth | When we multiply a number repeatedly by the same number $(\neq 1)$, resulting in the number increasing by the same proportion each time. <br> The original amount can grow very quickly in exponential growth. | $1,2,4,8,16,32,64,128 \ldots$ is an example of exponential growth, because the numbers are being multiplied by 2 each time. |
| 2. Exponential Decay | When we multiply a number repeatedly by the same number $(0<x<1)$, resulting in the number decreasing by the same proportion each time. <br> The original amount can decrease very quickly in exponential decay. | $1000,200,40,8 \ldots$ is an example of exponential decay, because the numbers are being multiplied by $\frac{1}{5}$ each time. |
| 3. Compound Interest | Interest paid on the original amount and the accumulated interest. | A bank pays 5\% compound interest a year. Bob invests $£ 3000$. How much will he have after 7 years. $3000 \times 1.05^{7}=£ 4221.30$ |
| 4. Exponential Graph | The equation is of the form $\boldsymbol{y}=\boldsymbol{a}^{\boldsymbol{x}}$, where $\boldsymbol{a}$ is a number called the base. <br> If $\boldsymbol{a}>\mathbf{1}$ the graph increases. <br> If $\mathbf{0}<\boldsymbol{a}<\mathbf{1}$, the graph decreases. <br> The graph has an asymptote which is the $\mathbf{x}$-axis. <br> The $\mathbf{y}$-intercept of the graph $y=a^{x}$ is $(0,1) \mathrm{s}$ |  |


| Topic/Skill | Definition/Tips | Example |
| :---: | :---: | :---: |
| 1. Real Life Graphs | Graphs that are supposed to model some real-life situation. <br> The actual meaning of the values depends on the labels and units on each axis. <br> The gradient might have a contextual meaning. <br> The $\mathbf{y}$-intercept might have a contextual meaning. <br> The area under the graph might have a contextual meaning. |  <br> A graph showing the cost of hiring a ladder for various numbers of days. <br> The gradient shows the cost per day. It costs $£ 3 /$ day to hire the ladder. <br> The y-intercept shows the additional cost/deposit/fixed charge (something not linked to how long the ladder is hired for). The additional cost is $£ 7$. |
| 2. Conversion Graph | A line graph to convert one unit to another. <br> Can be used to convert units (eg. miles and kilometres) or currencies (\$ and £) <br> Find the value you know on one axis, read up/across to the conversion line and read the equivalent value from the other axis. | Conversion graph miles $\longleftrightarrow$ kilometres <br> $8 \mathrm{~km}=5$ miles |
| 3. Depth of Water in Containers | Graphs can be used to show how the depth of water changes as different shaped containers are filled with water at a constant rate. |  |

