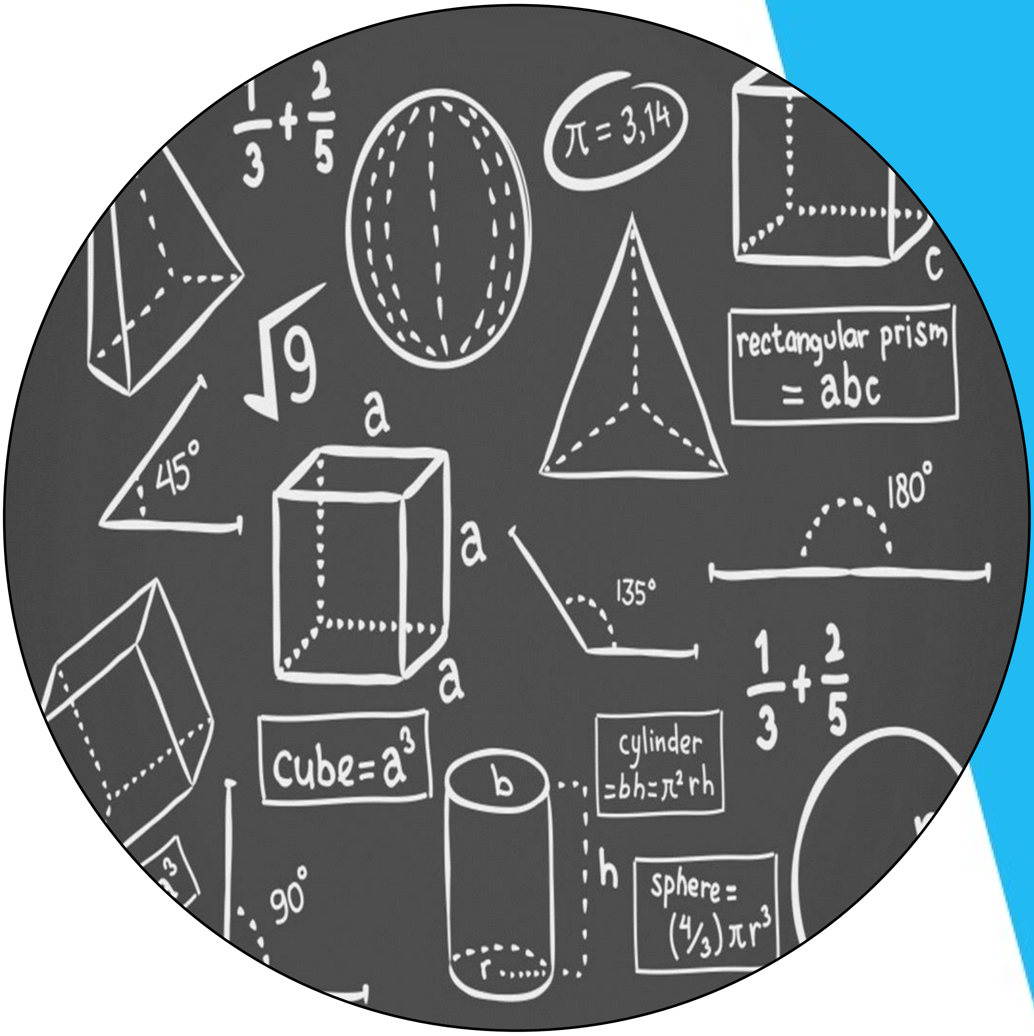
Knowledge Organiser



Mathematics – 11z3

RAYNES
PARK HIGH SCHOOL

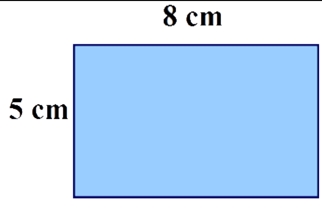
Topic: Basic Number and Decimals

Topic/Skill	Definition/Tips	Example
1. Integer	A **whole number** that can be positive, negative or zero.	$-3, 0, 92$
2. Decimal	A number with a **decimal point** in it. Can be positive or negative.	$3.7, 0.94, -24.07$
3. Negative Number	A number that is **less than zero**. Can be decimals.	$-8, -2.5$
4. Addition	To find the **total**, or **sum**, of two or more numbers. 'add', 'plus', 'sum'	$3 + 2 + 7 = 12$
5. Subtraction	To find the **difference** between two numbers. To find out how many are left when some are taken away. 'minus', 'take away', 'subtract'	$10 - 3 = 7$
6. Multiplication	Can be thought of as **repeated addition**. 'multiply', 'times', 'product'	$3 \times 6 = 6 + 6 + 6 = 18$
7. Division	Splitting into equal parts or groups. The process of calculating the **number of times one number is contained within another one**. 'divide', 'share'	$20 \div 4 = 5$ $\frac{20}{4} = 5$
8. Remainder	The amount '**left over**' after dividing one integer by another.	The remainder of $20 \div 6$ is 2, because 6 divides into 20 exactly 3 times, with 2 left over.
9. BIDMAS	An acronym for the **order** you should do calculations in. BIDMAS stands for **'Brackets, Indices, Division, Multiplication, Addition and Subtraction'**. Indices are also known as 'powers' or 'orders'. With strings of division and multiplication, or strings of addition and subtraction, and no brackets, work from left to right.	$6 + 3 \times 5 = 21, not\ 45$ $5^2 = 25$, where the 2 is the index/power. $12 \div 4 \div 2 = 1.5, not\ 6$
10. Recurring Decimal	A decimal number that has **digits that repeat forever**. The part that repeats is usually shown by placing a dot above the digit that repeats, or	$\frac{1}{3} = 0.333 \ldots = 0.\dot{3}$ $\frac{1}{7} = 0.142857142857 \ldots = 0.\dot{1}4285\dot{7}$

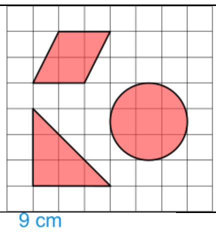
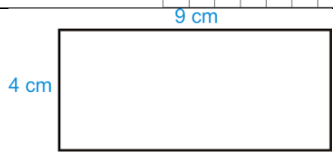
	dots over the first and last digit of the repeating pattern.	$\frac{77}{600} = 0.128333\ldots = 0.128\dot{3}$

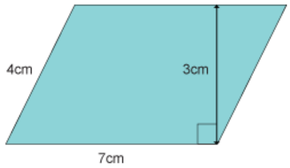
Topic: Perimeter and Area

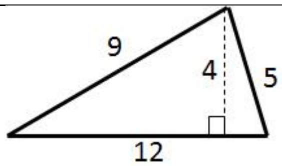
<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Perimeter</td><td>The total distance around the outside of a shape.

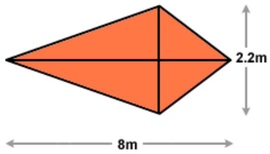
Units include: mm, cm, m etc.</td><td>

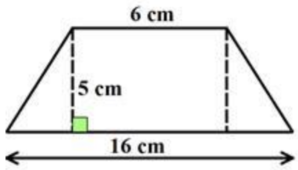
$P = 8 + 5 + 8 + 5 = 26cm$</td></tr>
<tr><td>2. Area</td><td>The amount of space inside a shape.

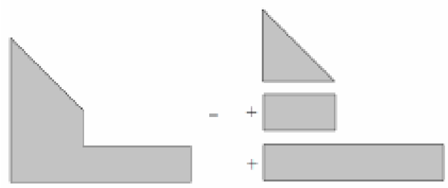
Units include: mm^2, cm^2, m^2</td><td></td></tr>
<tr><td>3. Area of a Rectangle</td><td>Length x Width</td><td>

$A = 36cm^2$</td></tr>
<tr><td>4. Area of a Parallelogram</td><td>Base x Perpendicular Height
Not the slant height.</td><td>

$A = 21cm^2$</td></tr>
<tr><td>5. Area of a Triangle</td><td>Base x Height ÷ 2</td><td>

$A = 24cm^2$</td></tr>
<tr><td>6. Area of a Kite</td><td>Split in to two triangles and use the method above.</td><td>

$A = 8.8m^2$</td></tr>
<tr><td>7. Area of a Trapezium</td><td>$$\frac{(a+b)}{2} \times h$$
“Half the sum of the parallel side, times the height between them. That is how you calculate the area of a trapezium”</td><td>

$A = 55cm^2$</td></tr>
<tr><td>8. Compound Shape</td><td>A shape made up of a combination of other known shapes put together.</td><td></td></tr>
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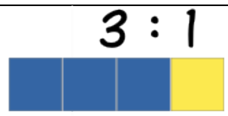
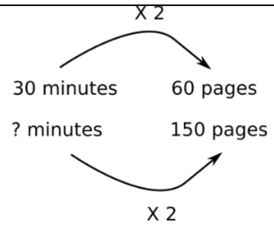
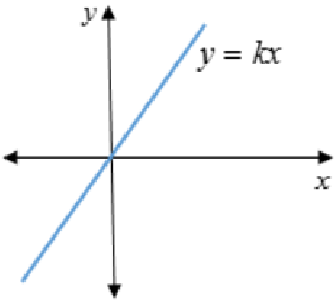
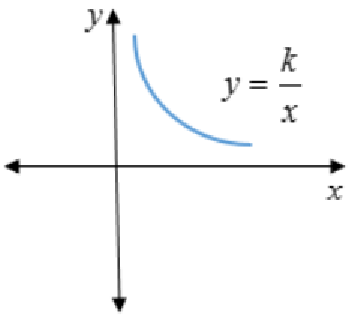
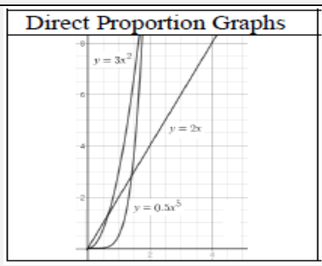
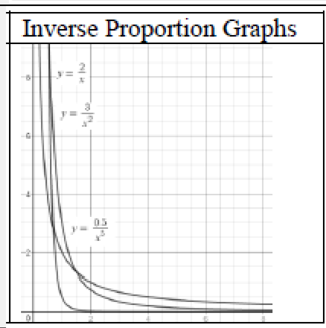
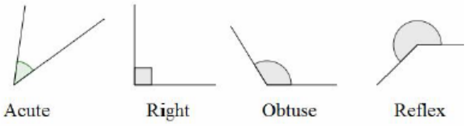
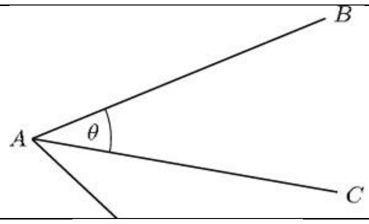
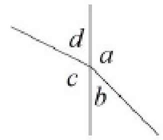
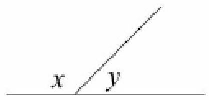
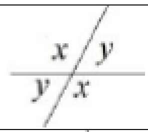
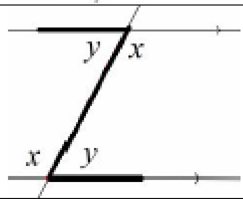
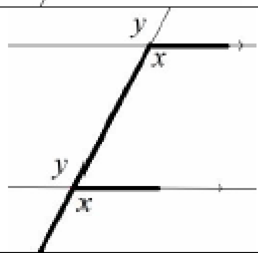
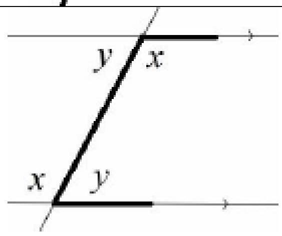
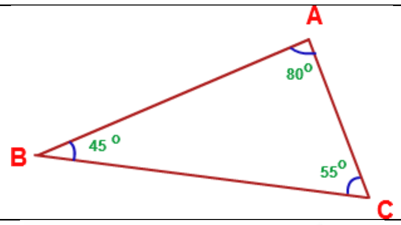
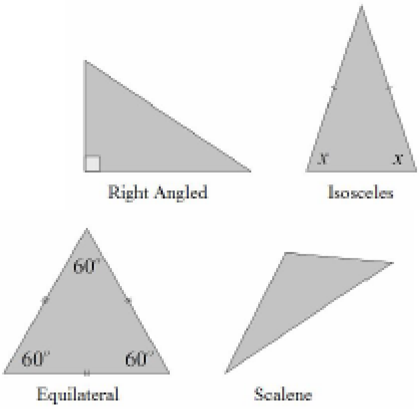
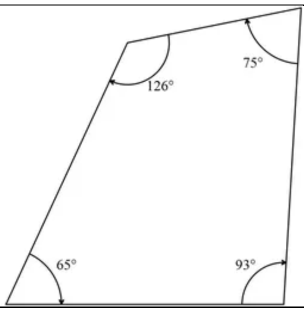
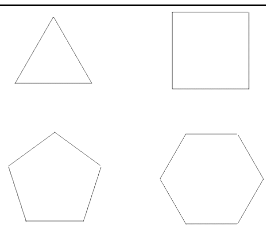
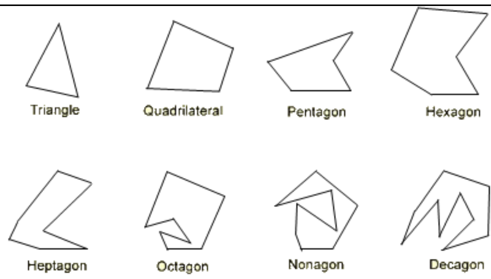
Topic: Ratio

Topic/Skill	Definition/Tips	Example
1. Ratio	Ratio compares the size of **one part** to **another part**. Written using the ':' symbol.	
2. Proportion	Proportion compares the size of **one part** to the size of the **whole**. Usually written as a fraction.	In a class with 13 boys and 9 girls, the proportion of boys is $\frac{13}{22}$ and the proportion of girls is $\frac{9}{22}$
3. Simplifying Ratios	**Divide** all parts of the ratio by a **common factor**.	5 : 10 = 1 : 2 (divide both by 5) 14 : 21 = 2 : 3 (divide both by 7)
4. Ratios in the form 1 : n or n : 1	**Divide** both parts of the ratio by one of the numbers to make **one part equal 1**.	$5 : 7 = 1 : \frac{7}{5}$ in the form 1 : n $5 : 7 = \frac{5}{7} : 1$ in the form n : 1
5. Sharing in a Ratio	**1. Add** the total parts of the ratio. **2. Divide** the amount to be shared by this value to find the value of one part. **3. Multiply** this value by each part of the ratio. Use only if you **know the total**.	Share £60 in the ratio 3 : 2 : 1. 3 + 2 + 1 = 6 60 ÷ 6 = 10 3 x 10 = 30, 2 x 10 = 20, 1 x 10 = 10 £30 : £20 : £10
6. Proportional Reasoning	Comparing two things using **multiplicative reasoning** and applying this to a new situation. Identify one multiplicative link and use this to find missing quantities.	
7. Unitary Method	Finding the **value of a single unit** and then finding the necessary value by **multiplying** the single unit value.	3 cakes require 450g of sugar to make. Find how much sugar is needed to make 5 cakes. 3 cakes = 450g So 1 cake = 150g (÷ by 3) So 5 cakes = 750 g (x by 5)
8. Ratio already shared	Find what **one part** of the ratio is worth using the **unitary method**.	Money was shared in the ratio 3:2:5 between Ann, Bob and Cat. Given that Bob had £16, found out the total amount of money shared. £16 = 2 parts So £8 = 1 part 3 + 2 + 5 = 10 parts, so 8 x 10 = £80
9. Best Buys	Find the **unit cost** by **dividing** the **price by the quantity**. The **lowest** number is the best value.	8 cakes for £1.28 → 16p each (÷by 8) 13 cakes for £2.05 → 15.8p each (÷by 13) Pack of 13 cakes is best value.

Topic: Proportion

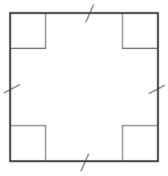
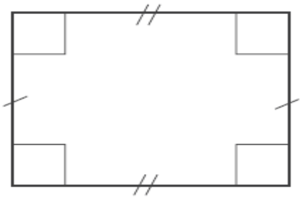
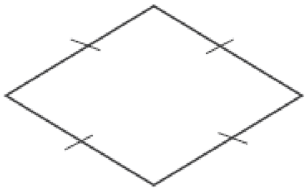
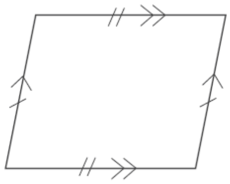
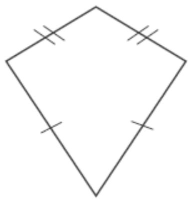
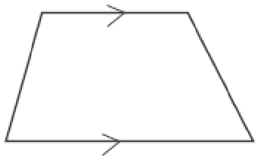
Topic/Skill	Definition/Tips	Example
1. Direct Proportion	If two quantities are in direct proportion, **as one increases**, the **other increases** by the **same percentage**. If y is directly proportional to x, this can be written as $\boldsymbol{y \propto x}$ An equation of the form $\boldsymbol{y = kx}$ represents direct proportion, where k **is the constant of proportionality**.	
2. Inverse Proportion	If two quantities are inversely proportional, **as one increases**, the **other decreases** by the **same percentage**. If y is inversely proportional to x, this can be written as $\boldsymbol{y \propto \frac{1}{x}}$ An equation of the form $\boldsymbol{y = \frac{k}{x}}$ represents inverse proportion.	
3. Using proportionality formulae	**Direct**: **y = kx** or **y ∝ x** **Inverse**: **y =** $\boldsymbol{\frac{k}{x}}$ or **y ∝** $\boldsymbol{\frac{1}{x}}$ 1. **Solve to find k** using the pair of values in the question. 2. **Rewrite the equation** using the k you have just found. 3. **Substitute the other given value** from the question in to the equation to **find the missing value**.	p is directly proportional to q. When p = 12, q = 4. Find p when q = 20. 1. p = kq 12 = k x 4 so k = 3 2. p = 3q 3. p = 3 x 20 = 60, so p = 60
4. Direct Proportion with powers	Graphs showing **direct proportion** can be written in the form $\boldsymbol{y = kx^n}$ Direct proportion graphs will always start at the origin.	
5. Inverse Proportion with powers	Graphs showing **inverse proportion** can be written in the form $\boldsymbol{y = \frac{k}{x^n}}$ Inverse proportion graphs will never start at the origin.	

Topic/Skill	Definition/Tips	Example
1. Types of Angles	**Acute angles** are less than 90°. **Right angles** are exactly 90°. **Obtuse angles** are greater than 90° but less than 180°. **Reflex angles** are greater than 180° but less than 360°.	
2. Angle Notation	Can use **one lower-case** letters, eg. θ or x Can use **three upper-case** letters, eg. BAC	
3. Angles at a Point	**Angles around a point add up to 360°**.	 $a+b+c+d=360^o$
4. Angles on a Straight Line	**Angles around a point on a straight line add up to 180°**.	 $x+y=180^o$
5. Opposite Angles	**Vertically opposite angles are equal**.	
6. Alternate Angles	**Alternate angles are equal**. They look like Z angles, but never say this in the exam.	
7. Corresponding Angles	**Corresponding angles are equal**. They look like F angles, but never say this in the exam.	
8. Co-Interior Angles	**Co-Interior angles add up to 180°**. They look like C angles, but never say this in the exam.	

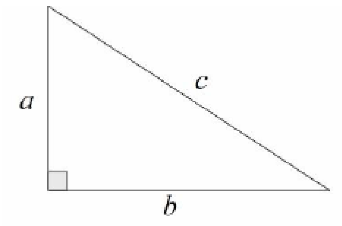
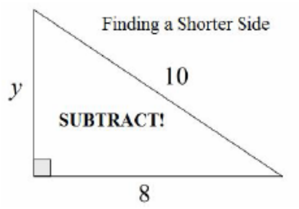
9. Angles in a Triangle	**Angles in a triangle add up to 180°.**	
10. Types of Triangles	**Right Angle** Triangles have a **90°** angle in. **Isosceles** Triangles have **2 equal sides** and **2 equal base angles**. **Equilateral** Triangles have **3 equal sides** and **3 equal angles (60°)**. **Scalene** Triangles have **different sides** and **different angles**. **Base angles in an isosceles triangle are equal.**	
11. Angles in a Quadrilateral	**Angles in a quadrilateral add up to 360°.**	
12. Polygon	A **2D** shape with **only straight edges**.	Rectangle, Hexagon, Decagon, Kite etc.
13. Regular	A shape is regular if all the **sides** and all the **angles** are **equal**.	
14. Names of Polygons	**3**-sided = **Triangle** **4**-sided = **Quadrilateral** **5**-sided = **Pentagon** **6**-sided = **Hexagon** **7**-sided = **Heptagon**/Septagon **8**-sided = **Octagon** **9**-sided = **Nonagon** **10**-sided = **Decagon**	
15. Sum of Interior Angles	$\boldsymbol{(n-2)\times 180}$ where n is the number of sides.	Sum of Interior Angles in a Decagon = $(10-2)\times 180 = 1440°$
16. Size of Interior Angle in a Regular Polygon	$\boldsymbol{\frac{(n-2)\times 180}{n}}$ You can also use the formula:	Size of Interior Angle in a Regular Pentagon = $\frac{(5-2)\times 180}{5} = 108°$

	$\mathbf{180} - \boldsymbol{Size\ of\ Exterior\ Angle}$	
17. Size of Exterior Angle in a Regular Polygon	$\frac{\mathbf{360}}{\boldsymbol{n}}$ You can also use the formula: $\mathbf{180} - \boldsymbol{Size\ of\ Interior\ Angle}$	Size of Exterior Angle in a Regular Octagon = $\frac{360}{8} = 45°$

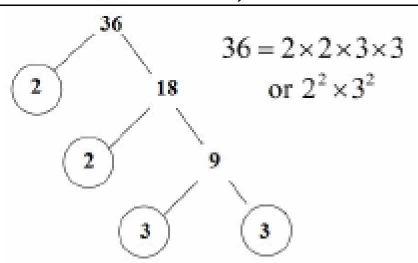
Topic: Properties of Polygons

Topic/Skill	Definition/Tips	Example
1. Square	• **Four equal sides** • **Four right angles** • **Opposite sides parallel** • **Diagonals bisect** each other at **right angles** • **Four lines** of **symmetry** • **Rotational symmetry** of **order four**	
2. Rectangle	• **Two pairs** of **equal sides** • **Four right angles** • **Opposite sides parallel** • **Diagonals bisect** each other**, not at right angles** • **Two lines** of **symmetry** • **Rotational symmetry** of **order two**	
3. Rhombus	• **Four equal sides** • **Diagonally opposite angles are equal** • **Opposite sides parallel** • **Diagonals bisect** each other at **right angles** • **Two lines** of **symmetry** • **Rotational symmetry** of **order two**	
4. Parallelogram	• **Two pairs** of **equal sides** • **Diagonally opposite angles are equal** • **Opposite sides parallel** • **Diagonals bisect** each other**, not at right angles** • **No lines** of **symmetry** • **Rotational symmetry** of **order two**	
5. Kite	• **Two pairs** of **adjacent sides** of **equal** length • **One pair** of **diagonally opposite angles are equal** (where different length sides meet) • **Diagonals intersect** at **right angles, but do not bisect** • **One line** of **symmetry** • **No rotational symmetry**	
6. Trapezium	• **One pair** of **parallel sides** • **No lines of symmetry** • **No rotational symmetry** Special Case: Isosceles Trapeziums have one line of symmetry.	

Topic: Pythagoras' Theorem

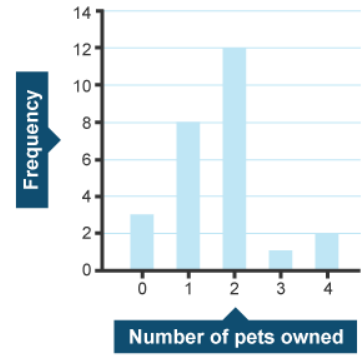
Topic/Skill	Definition/Tips	Example
1. Pythagoras' Theorem	For any **right angled triangle**: $$\boldsymbol{a^2 + b^2 = c^2}$$  Used to find **missing lengths**. a and b are the shorter sides, c is the **hypotenuse** (**longest side**).	 $a = y,\ b = 8,\ c = 10$ $a^2 = c^2 - b^2$ $y^2 = 100 - 64$ $y^2 = 36$ $y = 6$
2. 3D Pythagoras' Theorem	Find missing lengths by **identifying right angled triangles**. You will often have to find a missing length you are not asked for before finding the missing length you are asked for.	Can a pencil that is 20cm long fit in a pencil tin with dimensions 12cm, 13cm and 9cm? The pencil tin is in the shape of a cuboid. Hypotenuse of the base = $\sqrt{12^2 + 13^2} = 17.7$ Diagonal of cuboid = $\sqrt{17.7^2 + 9^2} = 19.8cm$ No, the pencil cannot fit.

Topic: Factors and Multiples

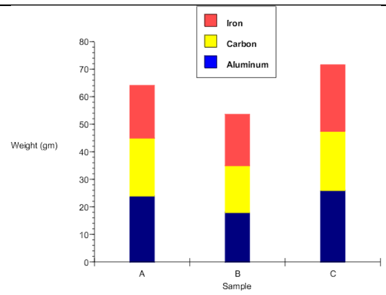
Topic/Skill	**Definition/Tips**	**Example**
1. Multiple	The result of multiplying a number by an integer. The **times tables** of a number.	The first five multiples of 7 are: $7, 14, 21, 28, 35$
2. Factor	A number that **divides exactly** into another number without a remainder. It is useful to write factors in pairs	The factors of 18 are: $1, 2, 3, 6, 9, 18$ The factor pairs of 18 are: $1, 18$ $2, 9$ $3, 6$
3. Lowest Common Multiple (LCM)	The **smallest** number that is in the **times tables** of each of the numbers given.	The LCM of 3, 4 and 5 is 60 because it is the smallest number in the 3, 4 and 5 times tables.
4. Highest Common Factor (HCF)	The **biggest** number that **divides exactly** into two or more numbers.	The HCF of 6 and 9 is 3 because it is the biggest number that divides into 6 and 9 exactly.
5. Prime Number	A number with **exactly two factors**. A number that can only be divided by itself and one. The number **1 is not prime**, as it only has one factor, not two.	The first ten prime numbers are: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$
6. Prime Factor	A factor which is a prime number.	The prime factors of 18 are: $2, 3$
7. Product of Prime Factors	Finding out which **prime numbers multiply** together to make the **original** number. Use a **prime factor tree.** Also known as ‘prime factorisation’.	

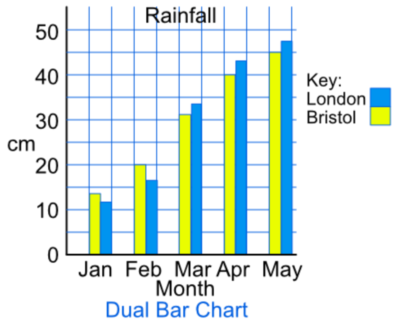
Topic: Representing Data

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Frequency Table</td><td>A record of how often each value in a set of data occurs.</td><td>
<table>
<tr><th>Number of marks</th><th>Tally marks</th><th>Frequency</th></tr>
<tr><td>1</td><td>卌 ||</td><td>7</td></tr>
<tr><td>2</td><td>卌</td><td>5</td></tr>
<tr><td>3</td><td>卌 |</td><td>6</td></tr>
<tr><td>4</td><td>卌</td><td>5</td></tr>
<tr><td>5</td><td>|||</td><td>3</td></tr>
<tr><td>Total</td><td></td><td>26</td></tr>
</table>
</td></tr>
<tr><td>2. Bar Chart</td><td>Represents data as vertical blocks.

$x - axis$ shows the type of data
$y - axis$ shows the frequency for each type of data
Each bar should be the same width
There should be gaps between each bar
Remember to label each axis.</td><td>


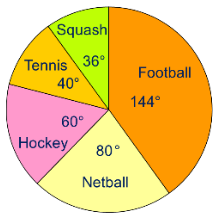
</td></tr>
<tr><td>3. Types of Bar Chart</td><td>Compound/Composite Bar Charts show data stacked on top of each other.

Comparative/Dual Bar Charts show data side by side.</td><td>




</td></tr>
<tr><td>4. Pie Chart</td><td>Used for showing how data breaks down into its constituent parts.

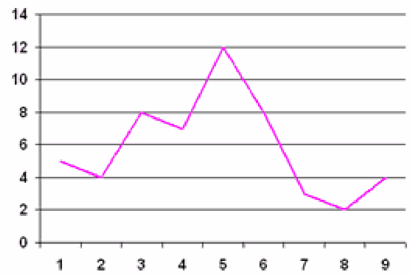
When drawing a pie chart, divide 360° by the total frequency. This will tell you how many degrees to use for the frequency of each category.

Remember to label the category that each sector in the pie chart represents.</td><td>


If there are 40 people in a survey, then each person will be worth 360÷40=9° of the pie chart.</td></tr>
</table>

<table>
<tr>
<td>5. Pictogram</td>
<td>Uses pictures or symbols to show the value of the data.

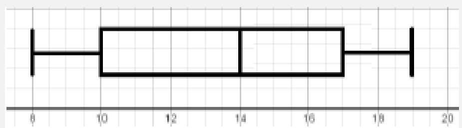
A pictogram must have a key.</td>
<td>
</td>
</tr>
<tr>
<td>6. Line Graph</td>
<td>A graph that uses points connected by straight lines to show how data changes in values.

This can be used for time series data, which is a series of data points spaced over uniform time intervals in time order.</td>
<td>
</td>
</tr>
<tr>
<td>7. Two Way Tables</td>
<td>A table that organises data around two categories.

Fill out the information step by step using the information given.

Make sure all the totals add up for all columns and rows.</td>
<td>Question: Complete the 2 way table below.
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td></td><td>58</td></tr>
<tr><td>Girls</td><td></td><td></td><td></td></tr>
<tr><td>Total</td><td></td><td>84</td><td>100</td></tr>
</table>
Answer: Step 1, fill out the easy parts (the totals)
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td>48</td><td>58</td></tr>
<tr><td>Girls</td><td></td><td></td><td>42</td></tr>
<tr><td>Total</td><td>16</td><td>84</td><td>100</td></tr>
</table>
Answer: Step 2, fill out the remaining parts
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td>48</td><td>58</td></tr>
<tr><td>Girls</td><td>6</td><td>36</td><td>42</td></tr>
<tr><td>Total</td><td>16</td><td>84</td><td>100</td></tr>
</table></td>
</tr>
<tr>
<td>8. Box Plots</td>
<td>The minimum, lower quartile, median, upper quartile and maximum are shown on a box plot.

A box plot can be drawn independently or from a cumulative frequency diagram.</td>
<td>Students sit a maths test. The highest score is 19, the lowest score is 8, the median is 14, the lower quartile is 10 and the upper quartile is 17. Draw a box plot to represent this information.


</td>
</tr>
<tr>
<td>9. Comparing Box Plots</td>
<td>Write two sentences.
1. Compare the averages using the medians for two sets of data.
2. Compare the spread of the data using the range or IQR for two sets of data.

The <u>smaller</u> the range/IQR, the <u>more consistent</u> the data.

You must compare box plots in the context of the problem.</td>
<td>‘On average, students in class A were more successful on the test than class B because their median score was higher.’

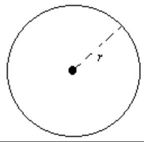
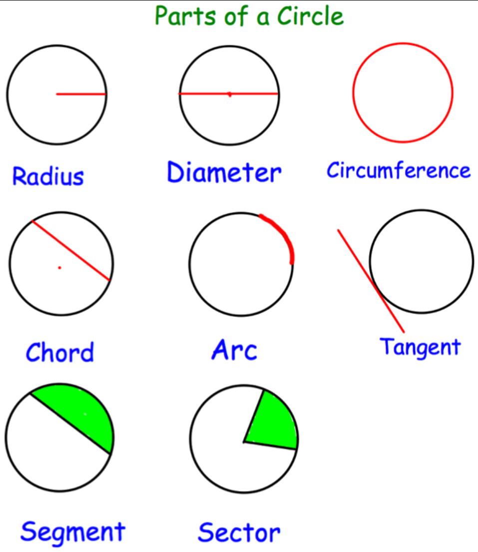
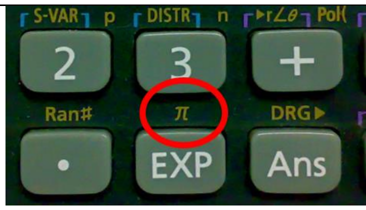
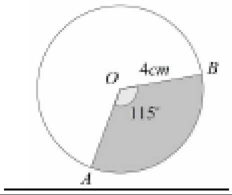
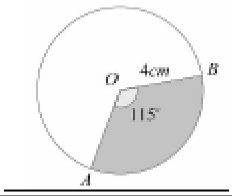
‘Students in class B were more consistent than class A in their test scores as their IQR was smaller.’</td>
</tr>
</table>

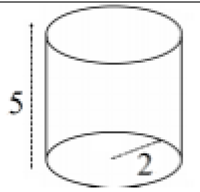
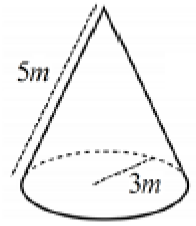
Topic: Indices

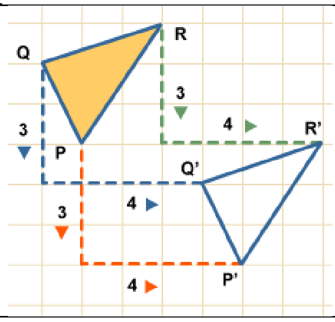
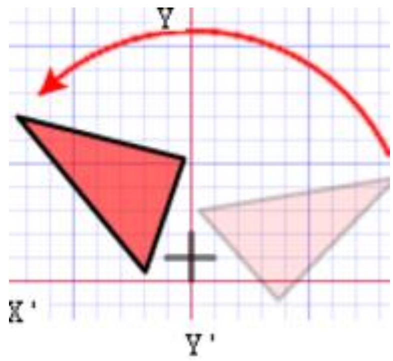
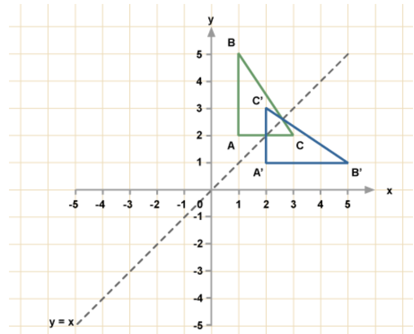
Topic/Skill	Definition/Tips	Example
1. Square Number	The number you get when you **multiply a number by itself**.	**1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225…** $9^2 = 9 \times 9 = 81$
2. Square Root	The **number you multiply by itself** to get another number. The reverse process of squaring a number.	$\sqrt{36} = 6$ because $6 \times 6 = 36$
3. Solutions to $x^2 =$ ….	**Equations** involving **squares** have **two solutions**, one **positive** and one **negative**.	Solve $x^2 = 25$ $x = 5 \text{ or } x = -5$ This can also be written as $x = \pm 5$
4. Cube Number	The number you get when you **multiply a number by itself and itself again**.	1, 8, 27, 64, 125… $2^3 = 2 \times 2 \times 2 = 8$
5. Cube Root	The **number you multiply by itself and itself again** to get another number. The reverse process of cubing a number.	$\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 125$
6. Powers of…	The powers of a number are that **number raised to various powers**.	The powers of 3 are: $3^1 = 3$ $3^2 = 9$ $3^3 = 27$ $3^4 = 81$ etc.
7. Multiplication Index Law	When **multiplying** with the same base (number or letter), **add the powers**. $\boldsymbol{a^m \times a^n = a^{m+n}}$	$7^5 \times 7^3 = 7^8$ $a^{12} \times a = a^{13}$ $4x^5 \times 2x^8 = 8x^{13}$
8. Division Index Law	When **dividing** with the same base (number or letter), **subtract the powers**. $\boldsymbol{a^m \div a^n = a^{m-n}}$	$15^7 \div 15^4 = 15^3$ $x^9 \div x^2 = x^7$ $20a^{11} \div 5a^3 = 4a^8$
9. Brackets Index Laws	When raising a power to another power, multiply the powers together. $\boldsymbol{(a^m)^n = a^{mn}}$	$(y^2)^5 = y^{10}$ $(6^3)^4 = 6^{12}$ $(5x^6)^3 = 125x^{18}$
10. Notable Powers	$\boldsymbol{p = p^1}$ $\boldsymbol{p^0 = 1}$	$99999^0 = 1$
11. Negative Powers	A negative power performs the reciprocal. $\boldsymbol{a^{-m} = \frac{1}{a^m}}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
12. Fractional Powers	The denominator of a fractional power acts as a 'root'. The numerator of a fractional power acts as a normal power. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$	$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$ $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{25}}{\sqrt{16}}\right)^3 = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$

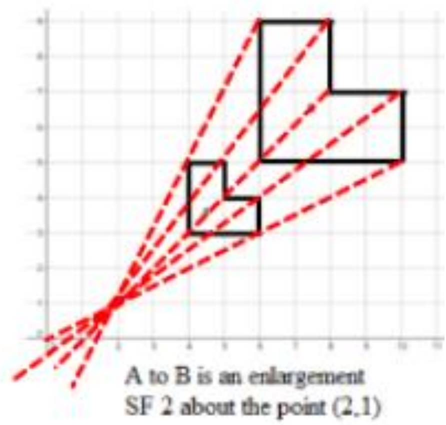
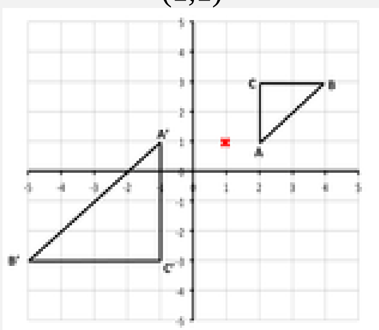
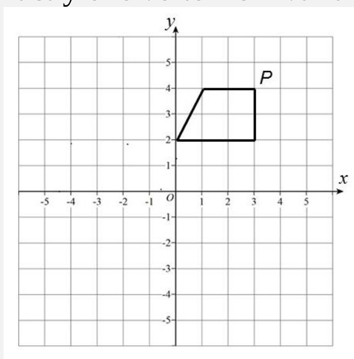
Topic: Standard Form

Topic/Skill	Definition/Tips	Example
1. Standard Form	$\boldsymbol{A \times 10^b}$ $where\ \mathbf{1} \le \boldsymbol{A} < \mathbf{10}, \quad \boldsymbol{b = integer}$	8400 = 8.4 x 10^3 0.00036 = 3.6 x 10^{-4}
2. Multiplying or Dividing with Standard Form	Multiply: **Multiply the numbers** and **add the powers**. Divide: **Divide the numbers** and **subtract the powers**.	$(1.2 \times 10^3) \times (4 \times 10^6) = 8.8 \times 10^9$ $(4.5 \times 10^5) \div (3 \times 10^2) = 1.5 \times 10^3$
3. Adding or Subtracting with Standard Form	**Convert** in to **ordinary** numbers, **calculate** and then **convert back** in to standard form	$2.7 \times 10^4 + 4.6 \times 10^3$ $= 27000 + 4600 = 31600$ $= 3.16 \times 10^4$

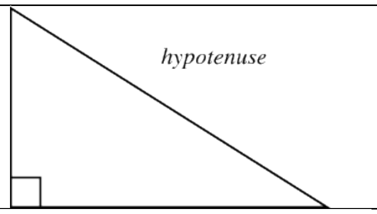
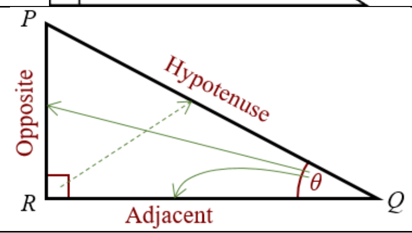
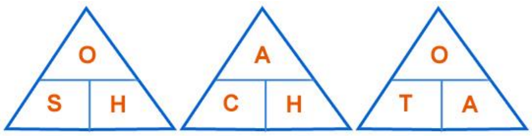
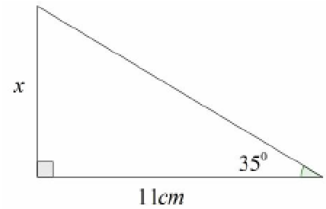
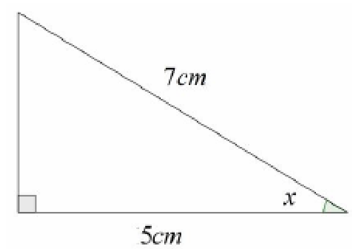
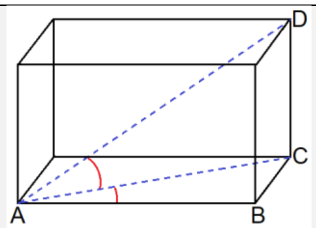
Topic/Skill	**Definition/Tips**	**Example**
1. Circle	A circle is the locus of all points equidistant from a central point.	
2. Parts of a Circle	**Radius** – the **distance** from the **centre** of a circle to the **edge** **Diameter** – the total **distance** across the **width** of a circle **through the centre**. **Circumference** – the **total distance** around the **outside** of a circle **Chord** – a **straight line** whose **end points lie on a circle** **Tangent** – a **straight line** which **touches** a circle at exactly **one point** **Arc** – a **part of the circumference** of a circle **Sector** – the **region** of a circle enclosed by **two radii** and their intercepted **arc** **Segment** – the **region** bounded by a **chord** and the **arc** created by the chord	
3. Area of a Circle	$\boldsymbol{A = \pi r^2}$ which means 'pi x radius squared'.	If the radius was 5cm, then: $A = \pi \times 5^2 = 78.5cm^2$
4. Circumference of a Circle	$\boldsymbol{C = \pi d}$ which means 'pi x diameter'	If the radius was 5cm, then: $C = \pi \times 10 = 31.4cm$
5. π ('pi')	Pi is the circumference of a circle divided by the diameter. $\boldsymbol{\pi \approx 3.14}$	
6. Arc Length of a Sector	The arc length is part of the circumference. Take the **angle** given **as a fraction over 360°** and **multiply** by the **circumference**.	Arc Length $= \frac{115}{360} \times \pi \times 8 = 8.03cm$ 
7. Area of a Sector	The area of a sector is part of the total area. Take the **angle** given **as a fraction over 360°** and **multiply** by the **area**.	Area $= \frac{115}{360} \times \pi \times 4^2 = 16.1cm^2$ 

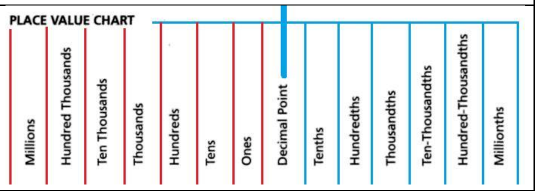
8. Surface Area of a Cylinder	**Curved Surface Area =** $\boldsymbol{\pi dh}$ or $\boldsymbol{2\pi rh}$ **Total SA =** $\boldsymbol{2\pi r^2 + \pi dh}$ or $\boldsymbol{2\pi r^2 + 2\pi rh}$	 $Total\ SA = 2\pi(2)^2 + \pi(4)(5) = 28\pi$
9. Surface Area of a Cone	**Curved Surface Area =** $\boldsymbol{\pi rl}$ where $l = slant\ height$ **Total SA =** $\boldsymbol{\pi rl + \pi r^2}$ You may need to use Pythagoras' Theorem to find the slant height	 $Total\ SA = \pi(3)(5) + \pi(3)^2 = 24\pi$
10. Surface Area of a Sphere	$\boldsymbol{SA = 4\pi r^2}$ Look out for hemispheres – halve the SA of a sphere and add on a circle (πr^2)	Find the surface area of a sphere with radius 3cm. $SA = 4\pi(3)^2 = 36\pi cm^2$

Topic/Skill	Definition/Tips	Example
1. Translation	**Translate** means to **move a shape**. The shape does not change **size** or **orientation**.	
2. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix}2\\3\end{pmatrix}$ means '2 right, 3 up' $\begin{pmatrix}-1\\-5\end{pmatrix}$ means '1 left, 5 down'
3. Rotation	The size does not change, but the **shape is turned around a point**. Use tracing paper.	Rotate Shape A 90° anti-clockwise about (0,1) 
4. Reflection	The size does not change, but the shape is '**flipped**' like in a **mirror**. Line x =? is a **vertical line**. Line y =? is a **horizontal line**. Line $y = x$ is a **diagonal line**.	Reflect shape C in the line $y = x$ 
5. Enlargement	The shape will get **bigger or smaller**. Multiply each side by the **scale factor**.	Scale Factor = 3 means '3 times larger = multiply by 3' Scale Factor = ½ means 'half the size = divide by 2'

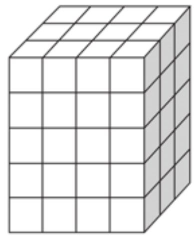
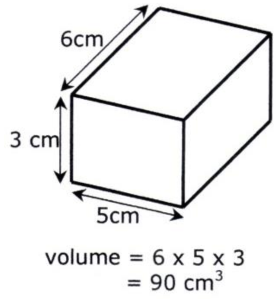
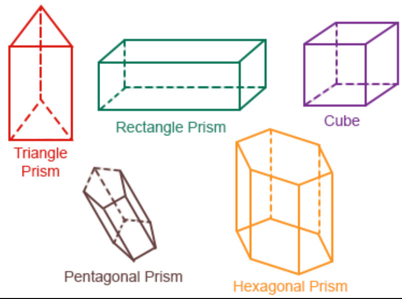
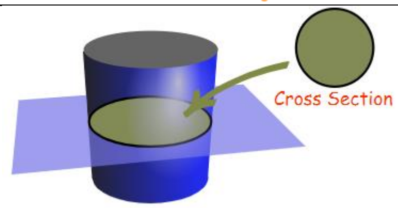
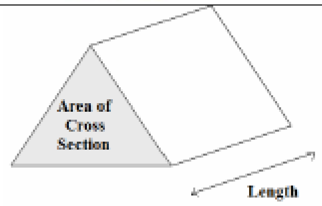
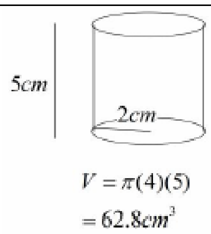
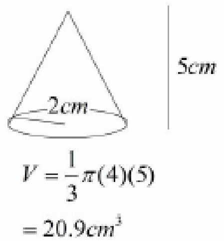
6. Finding the Centre of Enlargement	Draw **straight lines** through **corresponding corners** of the two shapes. The centre of enlargement is the point **where all the lines cross over**. Be careful with negative enlargements as the corresponding corners will be the other way around.	
7. Describing Transformations	Give the following information when describing each transformation: Look at the number of marks in the question for a hint of how many pieces of information are needed. If you are asked to describe a ‘transformation’, you need to say the **name of the type of transformation** as well as the other details.	**- Translation, Vector** **- Rotation, Direction, Angle, Centre** **- Reflection, Equation of mirror line** **- Enlargement, Scale factor, Centre of enlargement**
8. Negative Scale Factor Enlargements	Negative enlargements will **look like they have been rotated**. $SF = -2$ will be rotated, and also twice as big.	Enlarge ABC by scale factor -2, centre (1,1) 
9. Invariance	A point, line or shape is invariant if it **does not change/move** when a transformation is performed. An invariant point ‘does not vary’.	If shape P is reflected in the $y - axis$, then exactly one vertex is invariant. 

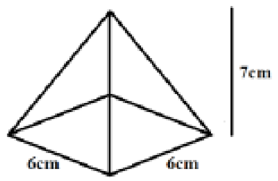
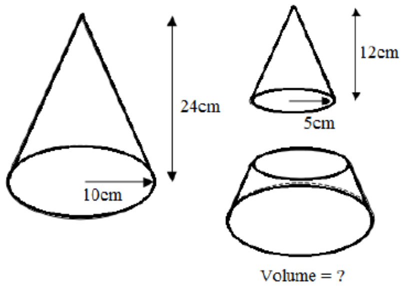
Topic: Right Angled Trigonometry

Topic/Skill	Definition/Tips	Example
1. Trigonometry	The **study** of **triangles**.	
2. Hypotenuse	The **longest side** of a **right-angled triangle**. Is always **opposite** the **right angle**.	
3. Adjacent	**Next to**	
4. Trigonometric Formulae	Use **SOHCAHTOA**. $\boldsymbol{\sin\theta = \frac{O}{H}}$ $\boldsymbol{\cos\theta = \frac{A}{H}}$ $\boldsymbol{\tan\theta = \frac{O}{A}}$  When finding a missing angle, use the 'inverse' trigonometric function by pressing the 'shift' button on the calculator.	 Use 'Opposite' and 'Adjacent', so use 'tan' $\tan 35 = \frac{x}{11}$ $x = 11\tan 35 = 7.70cm$  Use 'Adjacent' and 'Hypotenuse', so use 'cos' $\cos x = \frac{5}{7}$ $x = cos^{-1}\left(\frac{5}{7}\right) = 44.4°$
5. 3D Trigonometry	Find missing lengths by **identifying right angled triangles**. You will often have to find a missing length you are not asked for before finding the missing length you are asked for.	

Topic/Skill	**Definition/Tips**	**Example**
1. Place Value	The **value** of where a **digit** is within a number.	In 726, the value of the 2 is 20, as it is in the ‘tens’ column.
2. Place Value Columns	The names of the columns that **determine the value of each digit**. The ‘ones’ column is also known as the ‘units’ column.	
3. Rounding	To make a number simpler but keep its value close to what it was. If the **digit to the right** of the rounding digit is **less than 5, round down**. If the **digit to the right** of the rounding digit is **5 or more, round up**.	74 rounded to the nearest ten is 70, because 74 is closer to 70 than 80. 152,879 rounded to the nearest thousand is 153,000.
4. Decimal Place	The **position** of a digit to the **right of a decimal point**.	In the number 0.372, the 7 is in the second decimal place. 0.372 rounded to two decimal places is 0.37, because the 2 tells us to round down. Careful with money - don’t write £27.4, instead write £27.40
5. Significant Figure	The significant figures of a number are the digits which **carry meaning** (ie. are significant) to the size of the number. The **first significant figure** of a number **cannot be zero**. In a number with a decimal, trailing zeros are not significant.	In the number 0.00821, the first significant figure is the 8. In the number 2.740, the 0 is not a significant figure. 0.00821 rounded to 2 significant figures is 0.0082. 19357 rounded to 3 significant figures is 19400. We need to include the two zeros at the end to keep the digits in the same place value columns.
6. Truncation	A method of approximating a decimal number by **dropping all decimal places** past a certain point **without rounding**.	3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416)
7. Error Interval	A **range of values** that a number could have taken before being rounded or truncated. An error interval is written using inequalities, with a **lower bound** and an **upper bound**.	0.6 has been rounded to 1 decimal place. The error interval is: $0.55 \leq x < 0.65$ The lower bound is 0.55 The upper bound is 0.65

	Note that the lower bound inequality can be 'equal to', but the upper bound cannot be 'equal to'.	
8. Estimate	To find something **close to the correct answer**.	An estimate for the height of a man is 1.8 metres.
9. Approximation	When using approximations to estimate the solution to a calculation, **round each number in the calculation to 1 significant figure**. ≈ means 'approximately equal to'	$\frac{348+692}{0.526} \approx \frac{300+700}{0.5} = 2000$ 'Note that dividing by 0.5 is the same as multiplying by 2'
10. Rational Number	A number of the form $\boldsymbol{\frac{p}{q}}$**, where $\boldsymbol{p}$ and $\boldsymbol{q}$ are integers** and $\boldsymbol{q \neq 0}$. A number that cannot be written in this form is called an 'irrational' number	$\frac{4}{9}, 6, -\frac{1}{3}, \sqrt{25}$ are examples of rational numbers. $\pi, \sqrt{2}$ are examples of an irrational numbers.
11. Surd	The **irrational number** that is a **root of a positive integer,** whose value cannot be determined exactly. Surds have **infinite non-recurring decimals**.	$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly. $\sqrt{2} = 1.41421356 \ldots$ which never repeats.
12. Rules of Surds	$\boldsymbol{\sqrt{ab} = \sqrt{a} \times \sqrt{b}}$ $\boldsymbol{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$ $\boldsymbol{a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}}$ $\boldsymbol{\sqrt{a} \times \sqrt{a} = a}$	$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$ $2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$ $\sqrt{7} \times \sqrt{7} = 7$
13. Rationalise a Denominator	The process of rewriting a fraction so that the **denominator contains only rational numbers**.	$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6}}{2}$ $\frac{6}{3+\sqrt{7}} = \frac{6(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$ $= \frac{18-6\sqrt{7}}{9-7}$ $= \frac{18-6\sqrt{7}}{2} = 9 - 3\sqrt{7}$

Topic/Skill	Definition/Tips	Example
1. Volume	Volume is a measure of the amount of space inside a solid shape. Units: mm^3, cm^3, m^3 etc.	
2. Volume of a Cube/Cuboid	$V = Length \times Width \times Height$ $V = L \times W \times H$ You can also use the Volume of a Prism formula for a cube/cuboid.	
3. Prism	A prism is a 3D shape whose **cross section is the same** throughout.	
4. Cross Section	The **cross section** is the **shape** that **continues** all the way **through the prism**.	
5. Volume of a Prism	$V = Area\ of\ Cross\ Section \times Length$ $V = A \times L$	
6. Volume of a Cylinder	$V = \pi r^2 h$	
7. Volume of a Cone	$V = \frac{1}{3}\pi r^2 h$	

8. Volume of a Pyramid	$$\boldsymbol{Volume = \frac{1}{3}Bh}$$ where B = area of the base	 $$V = \frac{1}{3} \times 6 \times 6 \times 7 = 84cm^3$$
9. Volume of a Sphere	$$\boldsymbol{V = \frac{4}{3}\pi r^3}$$ Look out for hemispheres – just halve the volume of a sphere.	Find the volume of a sphere with diameter 10cm. $$V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3}cm^3$$
10. Frustums	A frustum is a solid (usually a cone or pyramid) with the **top removed**. Find the volume of the whole shape, then take away the volume of the small cone/pyramid removed at the top.	 $$V = \frac{1}{3}\pi(10)^2(24) - \frac{1}{3}\pi(5)^2(12)$$ $$= 700\pi cm^3$$

Topic: Fractions

Topic/Skill	Definition/Tips	Example
1. Fraction	A mathematical expression representing the **division** of one integer by another. Fractions are written as **two numbers separated by a horizontal line**.	$\frac{2}{7}$ is a 'proper' fraction. $\frac{9}{4}$ is an 'improper' or 'top-heavy' fraction.
2. Numerator	The **top** number of a fraction.	In the fraction $\frac{3}{5}$, 3 is the numerator.
3. Denominator	The **bottom** number of a fraction.	In the fraction $\frac{3}{5}$, 5 is the denominator.
4. Unit Fraction	A fraction where the **numerator is one** and the denominator is a positive integer.	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ $etc.$ are examples of unit fractions.
5. Reciprocal	The reciprocal of a number is **1 divided by the number**. The reciprocal of x is $\frac{1}{x}$ **When we multiply a number by its reciprocal we get 1**. This is called the 'multiplicative inverse'.	The reciprocal of 5 is $\frac{1}{5}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $$\frac{2}{3} \times \frac{3}{2} = 1$$
6. Mixed Number	A number formed of both an **integer part** and a **fraction part**.	$3\frac{2}{5}$ is an example of a mixed number.
7. Simplifying Fractions	**Divide the numerator and denominator by the highest common factor**.	$$\frac{20}{45} = \frac{4}{9}$$
8. Equivalent Fractions	Fractions which represent the **same value**.	$$\frac{2}{5} = \frac{4}{10} = \frac{20}{50} = \frac{60}{150}\ etc.$$
9. Comparing Fractions	To compare fractions, they each need to be rewritten so that they have a **common denominator**. **Ascending** means **smallest to biggest**. **Descending** means **biggest to smallest**.	Put in to ascending order : $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$. Equivalent: $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ Correct order: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
10. Fraction of an Amount	**Divide** by the **bottom**, **times** by the **top**	Find $\frac{2}{5}$ of £60 $$60 \div 5 = 12$$ $$12 \times 2 = 24$$
11. Adding or Subtracting Fractions	Find the **LCM of the denominators** to find a common denominator. Use equivalent fractions to change each fraction to the **common denominator**.	$$\frac{2}{3} + \frac{4}{5}$$ Multiples of 3: 3, 6, 9, 12, **15**.. Multiples of 5: 5, 10, **15**.. LCM of 3 and 5 = 15

	Then just **add or subtract the numerators** and keep the **denominator the same**.	$\frac{2}{3} = \frac{10}{15}$ $\frac{4}{5} = \frac{12}{15}$ $\frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}$
12. Multiplying Fractions	**Multiply** the **numerators** together and **multiply** the **denominators** together.	$\frac{3}{8} \times \frac{2}{9} = \frac{6}{72} = \frac{1}{12}$
13. Dividing Fractions	**'Keep it, Flip it, Change it – KFC'** Keep the first fraction the same Flip the second fraction upside down Change the divide to a multiply Multiply by the reciprocal of the second fraction.	$\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$

Topic: Basic Percentages

Topic/Skill	Definition/Tips	Example
1. Percentage	**Number of parts per 100.**	31% means $\frac{31}{100}$
2. Finding 10%	To find **10%**, **divide by 10**	10% of £36 = 36÷10=£3.60
3. Finding 1%	To find **1%**, **divide by 100**	1% of £8 = 8÷100 = £0.08
4. Percentage Change	$\frac{\boldsymbol{Difference}}{\boldsymbol{Original}} \times \mathbf{100}\%$	A games console is bought for £200 and sold for £250. % change = $\frac{50}{200} \times 100 = 25\%$
5. Fractions to Decimals	**Divide the numerator by the denominator** using the bus stop method.	$\frac{3}{8} = 3 \div 8 = 0.375$
6. Decimals to Fractions	**Write as a fraction** over 10, 100 or 1000 and simplify.	$0.36 = \frac{36}{100} = \frac{9}{25}$
7. Percentages to Decimals	**Divide by 100**	$8\% = 8 \div 100 = 0.08$
8. Decimals to Percentages	**Multiply by 100**	$0.4 = 0.4 \times 100\% = 40\%$
9. Fractions to Percentages	Percentage is just a fraction out of 100. **Make the denominator 100 using equivalent fractions**. When the denominator doesn't go in to 100, use a calculator and **multiply the fraction by 100**.	$\frac{3}{25} = \frac{12}{100} = 12\%$ $\frac{9}{17} \times 100 = 52.9\%$
10. Percentages to Fractions	Percentage is just a fraction out of 100. **Write the percentage over 100** and simplify.	$14\% = \frac{14}{100} = \frac{7}{50}$

Topic: Calculating with Percentages

Topic/Skill	Definition/Tips	Example
1. Increase or Decrease by a Percentage	Non-calculator: **Find the percentage** and **add** or **subtract** it from the **original** amount. Calculator: Find the **percentage multiplier** and multiply.	<u>Increase 500 by 20% (Non Calc):</u> 10% of 500 = 50 so 20% of 500 = 100 500 + 100 = 600 <u>Decrease 800 by 17% (Calc):</u> 100%-17%=83% 83% ÷ 100 = 0.83 0.83 x 800 = 664
2. Percentage Multiplier	The **number** you **multiply** a quantity by to **increase or decrease** it by a **percentage**.	The multiplier for increasing by 12% is 1.12 The multiplier for decreasing by 12% is 0.88 The multiplier for increasing by 100% is 2.
3. Reverse Percentage	Find the **correct percentage given in the question**, then work backwards to **find 100%** Look out for words like '**before**' or '**original**'	A jumper was priced at £48.60 after a 10% reduction. Find its original price. 100% - 10% = 90% 90% = £48.60 1% = £0.54 100% = £54
4. Simple Interest	Interest calculated as a **percentage of the original** amount.	£1000 invested for 3 years at 10% simple interest. 10% of £1000 = £100 Interest = $3 \times £100 = £300$

Topic: Algebra

Topic/Skill	Definition/Tips	Example
1. Expression	A mathematical statement written using **symbols**, **numbers** or **letters**,	$3x + 2$ or $5y^2$
2. Equation	A statement showing that **two expressions are equal**	$2y - 17 = 15$
3. Identity	An equation that is **true for all values** of the variables An identity uses the symbol: $\equiv$	$2x \equiv x+x$
4. Formula	Shows the **relationship** between **two or more variables**	Area of a rectangle = length x width or A= LxW
5. Simplifying Expressions	**Collect 'like terms'.** Be careful with negatives. x^2 and x are not like terms.	$2x + 3y + 4x - 5y + 3$ $= 6x - 2y + 3$ $3x + 4 - x^2 + 2x - 1 = 5x - x^2 + 3$
6. x times x	The answer is x^2 not $2x$.	Squaring is multiplying by itself, not by 2.
7. $p \times p \times p$	The answer is p^3 not $3p$	If p=2, then p^3=2x2x2=8, not 2x3=6
8. $p + p + p$	The answer is 3p not p^3	If p=2, then 2+2+2=6, not $2^3 = 8$
9. Expand	To expand a bracket, **multiply** each term **in the bracket** by the expression **outside** the bracket.	$3(m + 7) = 3x + 21$
10. Factorise	The **reverse** of **expanding**. Factorising is writing an expression as a product of terms by **'taking out' a common factor**.	$6x - 15 = 3(2x - 5)$, where 3 is the common factor.

Topic: Equations and Formulae

Topic/Skill	Definition/Tips	Example
1. Solve	To find the **answer**/value of something **Use inverse operations** on both sides of the equation (balancing method) until you find the value for the letter.	Solve $2x - 3 = 7$ Add 3 on both sides $2x = 10$ Divide by 2 on both sides $x = 5$
2. Inverse	**Opposite**	The inverse of addition is subtraction. The inverse of multiplication is division.
3. Rearranging Formulae	**Use inverse operations** on both sides of the formula (balancing method) until you find the expression for the letter.	Make x the subject of $y = \frac{2x-1}{z}$ Multiply both sides by z $yz = 2x - 1$ Add 1 to both sides $yz + 1 = 2x$ Divide by 2 on both sides $\frac{yz+1}{2} = x$ We now have x as the subject.
4. Writing Formulae	**Substitute letters for words** in the question.	Bob charges £3 per window and a £5 call out charge. $C = 3N + 5$ Where N=number of windows and C=cost
5. Substitution	**Replace letters with numbers**. Be careful of $5x^2$. You need to square first, then multiply by 5.	$a = 3, b = 2 \; and \; c = 5$. Find: 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$

Topic: Solving Quadratics by Factorising

Topic/Skill	**Definition/Tips**	**Example**
1. Quadratic	A quadratic expression is of the form $$\boldsymbol{ax^2 + bx + c}$$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions: $2x^3 - 5x^2$ $9x - 1$
2. Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2 - b^2}$ can be factorised to give $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b})$	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
4. Solving Quadratics $(ax^2 = b)$	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
5. Solving Quadratics $(ax^2 + bx = 0)$	**Factorise** and then **solve = 0**.	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0 \text{ or } x = 3$
6. Solving Quadratics by Factorising $(a = 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x + 5)(x - 2) = 0$ $x = -5 \text{ or } x = 2$
7. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
8. Solving Quadratics by Factorising $(a \neq 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2} \text{ or } x = -4$