Knowledge Organiser

Mathematics – 11z4

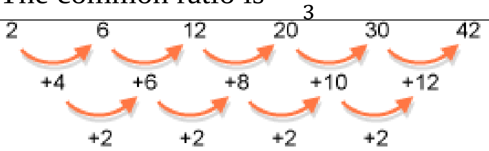
RAYNES
PARK HIGH SCHOOL

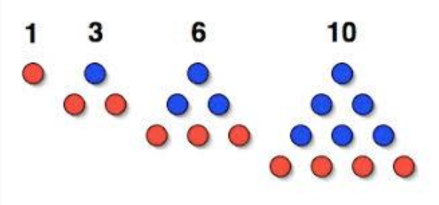
Topic: Basic Number and Decimals

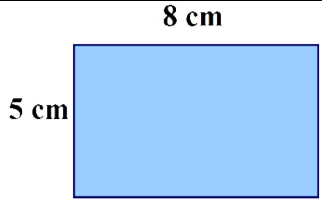
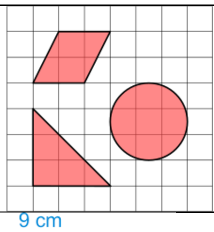
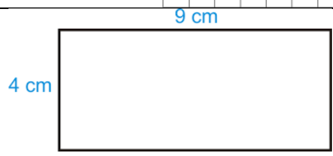
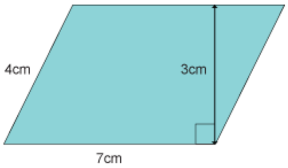
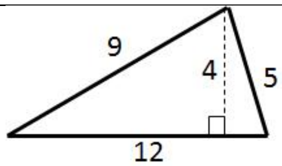
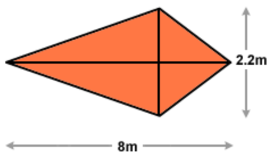
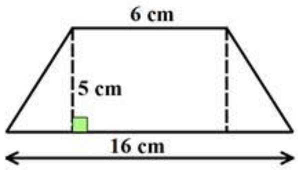
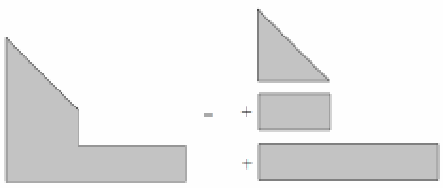
Topic/Skill	Definition/Tips	Example
1. Integer	A **whole number** that can be positive, negative or zero.	$-3, 0, 92$
2. Decimal	A number with a **decimal point** in it. Can be positive or negative.	$3.7, 0.94, -24.07$
3. Negative Number	A number that is **less than zero**. Can be decimals.	$-8, -2.5$
4. Addition	To find the **total**, or **sum**, of two or more numbers. 'add', 'plus', 'sum'	$3 + 2 + 7 = 12$
5. Subtraction	To find the **difference** between two numbers. To find out how many are left when some are taken away. 'minus', 'take away', 'subtract'	$10 - 3 = 7$
6. Multiplication	Can be thought of as **repeated addition**. 'multiply', 'times', 'product'	$3 \times 6 = 6 + 6 + 6 = 18$
7. Division	Splitting into equal parts or groups. The process of calculating the **number of times one number is contained within another one**. 'divide', 'share'	$20 \div 4 = 5$ $\frac{20}{4} = 5$
8. Remainder	The amount '**left over**' after dividing one integer by another.	The remainder of $20 \div 6$ is 2, because 6 divides into 20 exactly 3 times, with 2 left over.
9. BIDMAS	An acronym for the **order** you should do calculations in. BIDMAS stands for **'Brackets, Indices, Division, Multiplication, Addition and Subtraction'**. Indices are also known as 'powers' or 'orders'. With strings of division and multiplication, or strings of addition and subtraction, and no brackets, work from left to right.	$6 + 3 \times 5 = 21, not\ 45$ $5^2 = 25$, where the 2 is the index/power. $12 \div 4 \div 2 = 1.5, not\ 6$
10. Recurring Decimal	A decimal number that has **digits that repeat forever**. The part that repeats is usually shown by placing a dot above the digit that repeats, or	$\frac{1}{3} = 0.333 \ldots = 0.\dot{3}$ $\frac{1}{7} = 0.142857142857 \ldots = 0.\dot{1}4285\dot{7}$

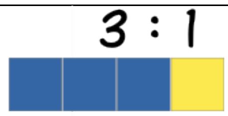
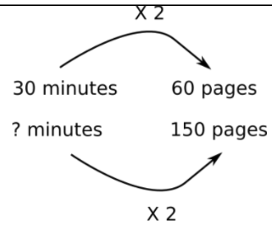
	dots over the first and last digit of the repeating pattern.	$\frac{77}{600} = 0.128333\ldots = 0.128\dot{3}$

Topic: Sequences

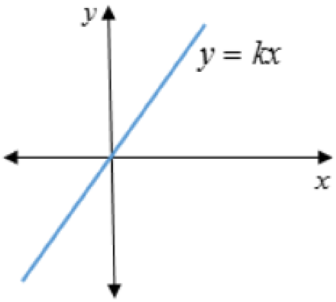
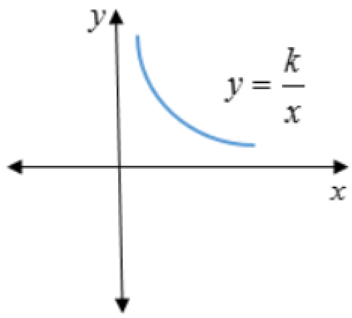
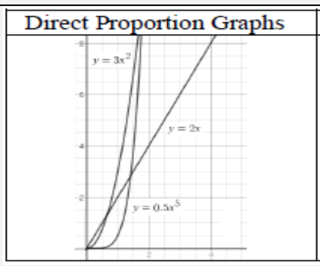
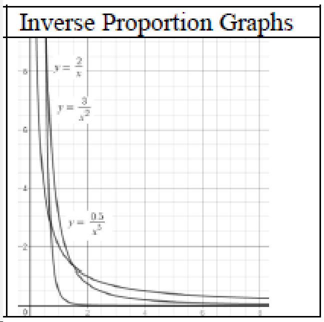
Topic/Skill	Definition/Tips	Example
1. Linear Sequence	A number pattern with a **common difference**.	2, 5, 8, 11… is a linear sequence
2. Term	**Each value** in a sequence is called a term.	In the sequence 2, 5, 8, 11…, 8 is the third term of the sequence.
3. Term-to-term rule	A rule which allows you to **find the next term** in a sequence if you **know the previous term**.	First term is 2. Term-to-term rule is 'add 3' Sequence is: 2, 5, 8, 11…
4. nth term	A rule which allows you to **calculate the term** that is in the **nth position** of the sequence. Also known as the 'position-to-term' rule. **n** refers to the **position** of a term in a sequence.	nth term is $3n - 1$ The 100th term is $3 \times 100 - 1 = 299$
5. Finding the nth term of a linear sequence	1. Find the **difference**. 2. **Multiply that by n.** 3. Substitute $n = 1$ to **find out what number you need to add or subtract to get the first number in the sequence**.	Find the nth term of: 3, 7, 11, 15… 1. Difference is +4 2. Start with $4n$ 3. $4 \times 1 = 4$, so we need to subtract 1 to get 3. nth term = $4n - 1$
6. Fibonacci type sequences	A sequence where the next number is found by **adding up the previous two terms**	The Fibonacci sequence is: $1,1,2,3,5,8,13,21,34$ … An example of a Fibonacci-type sequence is: $4, 7, 11, 18, 29$ …
7. Geometric Sequence	A sequence of numbers where each term is found by **multiplying the previous one** by a number called the **common ratio, r**.	An example of a geometric sequence is: $2, 10, 50, 250$ … The common ratio is 5 Another example of a geometric sequence is: $81, -27, 9, -3, 1$ … The common ratio is $-\frac{1}{3}$
8. Quadratic Sequence	A sequence of numbers where the **second difference is constant**. A quadratic sequence will have a n^2 term.	
9. nth term of a geometric sequence	$\boldsymbol{ar^{n-1}}$ where a is the first term and r is the common ratio	The nth term of $2, 10, 50, 250$ …. Is $2 \times 5^{n-1}$

10. nth term of a quadratic sequence	1. Find the first and second differences. 2. Halve the second difference and multiply this by n^2. 3. Substitute $n = 1,2,3,4$ ... into your expression so far. 4. Subtract this set of numbers from the corresponding terms in the sequence from the question. 5. Find the nth term of this set of numbers. 6. Combine the nth terms to find the overall nth term of the quadratic sequence. Substitute values in to check your nth term works for the sequence.	Find the nth term of: 4, 7, 14, 25, 40.. Answer: Second difference = +4 → nth term = $2n^2$ Sequence: 4, 7, 14, 25, 40 $2n^2$ 2, 8, 18, 32, 50 Difference: 2, -1, -4, -7, -10 Nth term of this set of numbers is $-3n + 5$ Overall nth term: $2n^2 - 3n + 5$
11. Triangular numbers	The sequence which comes from a pattern of dots that form a triangle. $1, 3, 6, 10, 15, 21$ ...	

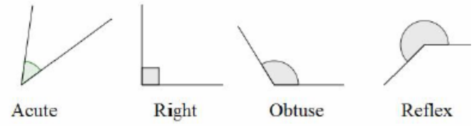
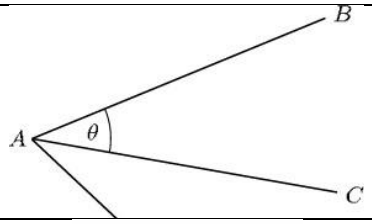
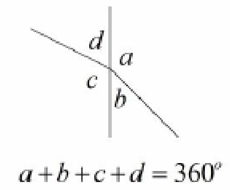
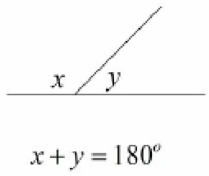
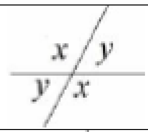
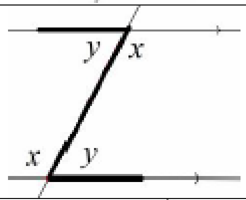
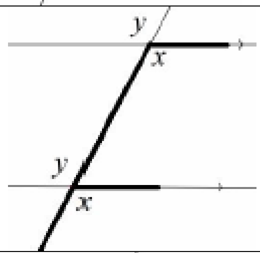
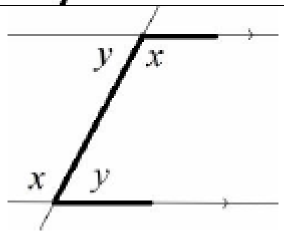
Topic/Skill	Definition/Tips	Example
1. Perimeter	The **total distance** around the **outside** of a shape. Units include: mm, cm, m etc.	 $P = 8 + 5 + 8 + 5 = 26cm$
2. Area	The amount of **space inside** a shape. Units include: mm^2, cm^2, m^2	
3. Area of a Rectangle	**Length x Width**	 $A = 36cm^2$
4. Area of a Parallelogram	**Base x Perpendicular Height** Not the slant height.	 $A = 21cm^2$
5. Area of a Triangle	**Base x Height ÷ 2**	 $A = 24cm^2$
6. Area of a Kite	Split in to **two triangles** and use the method above.	 $A = 8.8m^2$
7. Area of a Trapezium	$$\frac{(a+b)}{2} \times h$$ "Half the sum of the parallel side, times the height between them. That is how you calculate the area of a trapezium"	 $A = 55cm^2$
8. Compound Shape	A shape made up of a **combination of other known shapes** put together.	

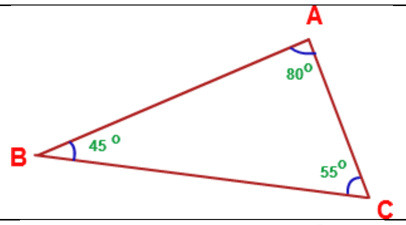
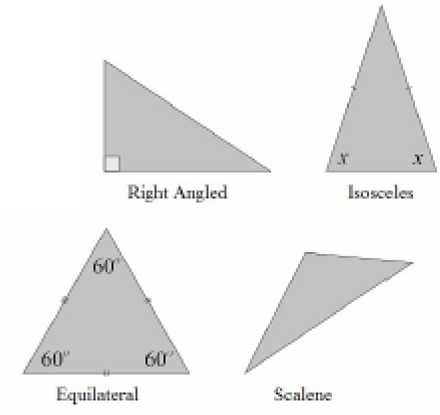
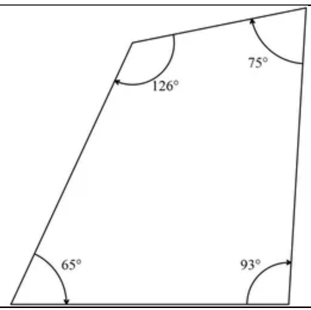
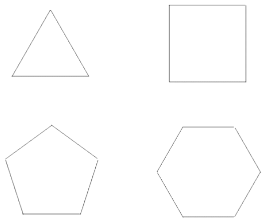
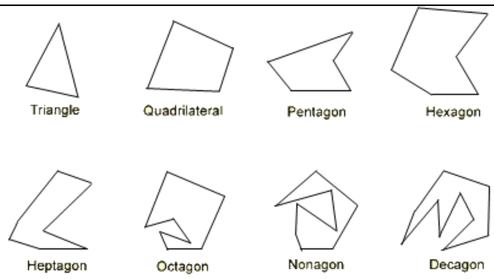
Topic/Skill	**Definition/Tips**	**Example**
1. Ratio	Ratio compares the size of **one part** to **another part**. Written using the ‘:’ symbol.	
2. Proportion	Proportion compares the size of **one part** to the size of the **whole**. Usually written as a fraction.	In a class with 13 boys and 9 girls, the proportion of boys is $\frac{13}{22}$ and the proportion of girls is $\frac{9}{22}$
3. Simplifying Ratios	**Divide** all parts of the ratio by a **common factor**.	5 : 10 = 1 : 2 (divide both by 5) 14 : 21 = 2 : 3 (divide both by 7)
4. Ratios in the form 1 : n or n : 1	**Divide** both parts of the ratio by one of the numbers to make **one part equal 1**.	5 : 7 = 1 : $\frac{7}{5}$ in the form 1 : n 5 : 7 = $\frac{5}{7}$: 1 in the form n : 1
5. Sharing in a Ratio	**1. Add** the total parts of the ratio. **2. Divide** the amount to be shared by this value to find the value of one part. **3. Multiply** this value by each part of the ratio. Use only if you **know the total**.	Share £60 in the ratio 3 : 2 : 1. 3 + 2 + 1 = 6 60 ÷ 6 = 10 3 x 10 = 30, 2 x 10 = 20, 1 x 10 = 10 £30 : £20 : £10
6. Proportional Reasoning	Comparing two things using **multiplicative reasoning** and applying this to a new situation. Identify one multiplicative link and use this to find missing quantities.	
7. Unitary Method	Finding the **value of a single unit** and then finding the necessary value by **multiplying** the single unit value.	3 cakes require 450g of sugar to make. Find how much sugar is needed to make 5 cakes. 3 cakes = 450g So 1 cake = 150g (÷ by 3) So 5 cakes = 750 g (x by 5)
8. Ratio already shared	Find what **one part** of the ratio is worth using the **unitary method**.	Money was shared in the ratio 3:2:5 between Ann, Bob and Cat. Given that Bob had £16, found out the total amount of money shared. £16 = 2 parts So £8 = 1 part 3 + 2 + 5 = 10 parts, so 8 x 10 = £80
9. Best Buys	Find the **unit cost** by **dividing** the **price by the quantity**. The **lowest** number is the best value.	8 cakes for £1.28 → 16p each (÷by 8) 13 cakes for £2.05 → 15.8p each (÷by 13) Pack of 13 cakes is best value.

Topic: Proportion

Topic/Skill	Definition/Tips	Example
1. Direct Proportion	If two quantities are in direct proportion, **as one increases**, the **other increases** by the **same percentage**. If y is directly proportional to x, this can be written as $\boldsymbol{y \propto x}$ An equation of the form $\boldsymbol{y = kx}$ represents direct proportion, where k **is the constant of proportionality**.	
2. Inverse Proportion	If two quantities are inversely proportional, **as one increases**, the **other decreases** by the **same percentage**. If y is inversely proportional to x, this can be written as $\boldsymbol{y \propto \frac{1}{x}}$ An equation of the form $\boldsymbol{y = \frac{k}{x}}$ represents inverse proportion.	
3. Using proportionality formulae	**Direct**: **y = kx** or **y ∝ x** **Inverse**: **y =** $\boldsymbol{\frac{k}{x}}$ or **y ∝** $\boldsymbol{\frac{1}{x}}$ 1. **Solve to find k** using the pair of values in the question. 2. **Rewrite the equation** using the k you have just found. 3. **Substitute the other given value** from the question in to the equation to **find the missing value**.	p is directly proportional to q. When p = 12, q = 4. Find p when q = 20. 1. p = kq 12 = k x 4 so k = 3 2. p = 3q 3. p = 3 x 20 = 60, so p = 60
4. Direct Proportion with powers	Graphs showing **direct proportion** can be written in the form $\boldsymbol{y = kx^n}$ Direct proportion graphs will always start at the origin.	
5. Inverse Proportion with powers	Graphs showing **inverse proportion** can be written in the form $\boldsymbol{y = \frac{k}{x^n}}$ Inverse proportion graphs will never start at the origin.	

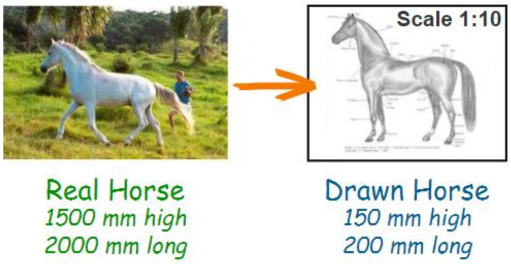
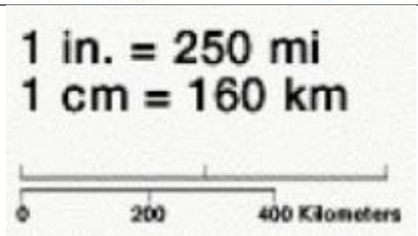
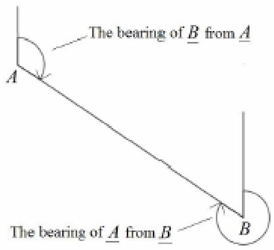
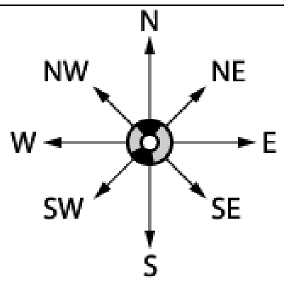
Topic: Angles

Topic/Skill	Definition/Tips	Example
1. Types of Angles	**Acute angles** are less than 90°. **Right angles** are exactly 90°. **Obtuse angles** are greater than 90° but less than 180°. **Reflex angles** are greater than 180° but less than 360°.	
2. Angle Notation	Can use **one lower-case** letters, eg. θ or x Can use **three upper-case** letters, eg. BAC	
3. Angles at a Point	**Angles around a point add up to 360°**.	
4. Angles on a Straight Line	**Angles around a point on a straight line add up to 180°**.	
5. Opposite Angles	**Vertically opposite angles are equal**.	
6. Alternate Angles	**Alternate angles are equal**. They look like Z angles, but never say this in the exam.	
7. Corresponding Angles	**Corresponding angles are equal**. They look like F angles, but never say this in the exam.	
8. Co-Interior Angles	**Co-Interior angles add up to 180°**. They look like C angles, but never say this in the exam.	

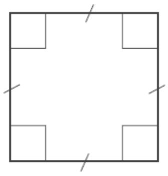
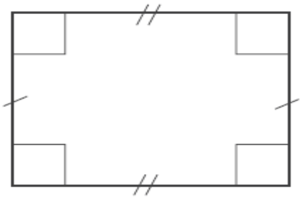
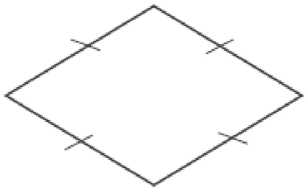
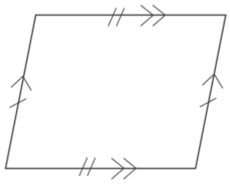
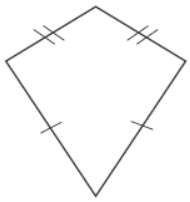
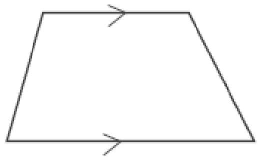
9. Angles in a Triangle	**Angles in a triangle add up to 180°.**	
10. Types of Triangles	**Right Angle** Triangles have a **90°** angle in. **Isosceles** Triangles have **2 equal sides** and **2 equal base angles**. **Equilateral** Triangles have **3 equal sides** and **3 equal angles (60°)**. **Scalene** Triangles have **different sides** and **different angles**. **Base angles in an isosceles triangle are equal.**	
11. Angles in a Quadrilateral	**Angles in a quadrilateral add up to 360°.**	
12. Polygon	A **2D** shape with **only straight edges**.	Rectangle, Hexagon, Decagon, Kite etc.
13. Regular	A shape is regular if all the **sides** and all the **angles** are **equal**.	
14. Names of Polygons	**3**-sided = **Triangle** **4**-sided = **Quadrilateral** **5**-sided = **Pentagon** **6**-sided = **Hexagon** **7**-sided = **Heptagon**/Septagon **8**-sided = **Octagon** **9**-sided = **Nonagon** **10**-sided = **Decagon**	
15. Sum of Interior Angles	$(\boldsymbol{n}-\mathbf{2})\times\mathbf{180}$ where n is the number of sides.	Sum of Interior Angles in a Decagon = $(10-2)\times 180 = 1440°$
16. Size of Interior Angle in a Regular Polygon	$\frac{(\boldsymbol{n}-\mathbf{2})\times\mathbf{180}}{\boldsymbol{n}}$ You can also use the formula:	Size of Interior Angle in a Regular Pentagon = $\frac{(5-2)\times 180}{5} = 108°$

	$\mathbf{180} - \boldsymbol{Size\ of\ Exterior\ Angle}$	
17. Size of Exterior Angle in a Regular Polygon	$\frac{\mathbf{360}}{\boldsymbol{n}}$ You can also use the formula: $\mathbf{180} - \boldsymbol{Size\ of\ Interior\ Angle}$	Size of Exterior Angle in a Regular Octagon = $\frac{360}{8} = 45°$

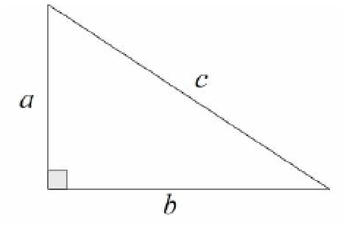
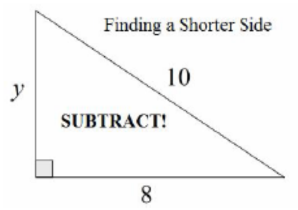
Topic: Bearings and Scale Diagrams

Topic/Skill	Definition/Tips	Example
1. Scale	The **ratio** of the **length** in a **model** to the length of the **real** thing.	
2. Scale (Map)	The **ratio** of a **distance on the map** to the actual **distance in real life**.	
3. Bearings	1. Measure from **North** (draw a North line) 2. Measure **clockwise** 3. Your answer must have **3 digits** (eg. 047°) Look out for where the bearing is measured <u>from</u>.	
4. Compass Directions	You can use an acronym such as **'Never Eat Shredded Wheat'** to remember the order of the compass directions in a clockwise direction. Bearings: $NE = 045°, W = 270°\ etc.$	

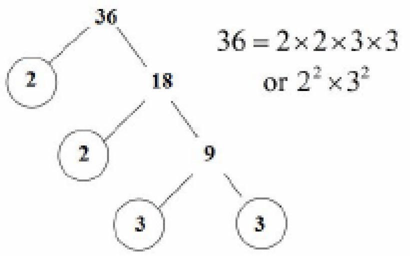
Topic: Properties of Polygons

Topic/Skill	Definition/Tips	Example
1. Square	• **Four equal sides** • **Four right angles** • **Opposite sides parallel** • **Diagonals bisect** each other at **right angles** • **Four lines** of **symmetry** • **Rotational symmetry** of **order four**	
2. Rectangle	• **Two pairs** of **equal sides** • **Four right angles** • **Opposite sides parallel** • **Diagonals bisect** each other**, not at right angles** • **Two lines** of **symmetry** • **Rotational symmetry** of **order two**	
3. Rhombus	• **Four equal sides** • **Diagonally opposite angles are equal** • **Opposite sides parallel** • **Diagonals bisect** each other at **right angles** • **Two lines** of **symmetry** • **Rotational symmetry** of **order two**	
4. Parallelogram	• **Two pairs** of **equal sides** • **Diagonally opposite angles are equal** • **Opposite sides parallel** • **Diagonals bisect** each other**, not at right angles** • **No lines** of **symmetry** • **Rotational symmetry** of **order two**	
5. Kite	• **Two pairs** of **adjacent sides** of **equal** length • **One pair** of **diagonally opposite angles are equal** (where different length sides meet) • **Diagonals intersect** at **right angles, but do not bisect** • **One line** of **symmetry** • **No rotational symmetry**	
6. Trapezium	• **One pair** of **parallel sides** • **No lines of symmetry** • **No rotational symmetry** Special Case: Isosceles Trapeziums have one line of symmetry.	

Topic: Pythagoras' Theorem

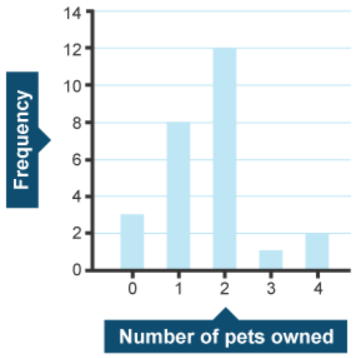
Topic/Skill	Definition/Tips	Example
1. Pythagoras' Theorem	For any **right angled triangle**: $$\boldsymbol{a^2 + b^2 = c^2}$$  Used to find **missing lengths**. a and b are the shorter sides, c is the **hypotenuse** (**longest side**).	 $a = y,\ b = 8,\ c = 10$ $a^2 = c^2 - b^2$ $y^2 = 100 - 64$ $y^2 = 36$ $y = 6$
2. 3D Pythagoras' Theorem	Find missing lengths by **identifying right angled triangles**. You will often have to find a missing length you are not asked for before finding the missing length you are asked for.	Can a pencil that is 20cm long fit in a pencil tin with dimensions 12cm, 13cm and 9cm? The pencil tin is in the shape of a cuboid. Hypotenuse of the base = $\sqrt{12^2 + 13^2} = 17.7$ Diagonal of cuboid = $\sqrt{17.7^2 + 9^2} = 19.8cm$ No, the pencil cannot fit.

Topic: Factors and Multiples

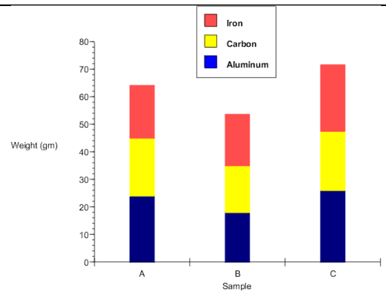
Topic/Skill	Definition/Tips	Example
1. Multiple	The result of multiplying a number by an integer. The **times tables** of a number.	The first five multiples of 7 are: $7, 14, 21, 28, 35$
2. Factor	A number that **divides exactly** into another number without a remainder. It is useful to write factors in pairs	The factors of 18 are: $1, 2, 3, 6, 9, 18$ The factor pairs of 18 are: $1, 18$ $2, 9$ $3, 6$
3. Lowest Common Multiple (LCM)	The **smallest** number that is in the **times tables** of each of the numbers given.	The LCM of 3, 4 and 5 is 60 because it is the smallest number in the 3, 4 and 5 times tables.
4. Highest Common Factor (HCF)	The **biggest** number that **divides exactly** into two or more numbers.	The HCF of 6 and 9 is 3 because it is the biggest number that divides into 6 and 9 exactly.
5. Prime Number	A number with **exactly two factors**. A number that can only be divided by itself and one. The number **1 is not prime**, as it only has one factor, not two.	The first ten prime numbers are: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$
6. Prime Factor	A factor which is a prime number.	The prime factors of 18 are: $2, 3$
7. Product of Prime Factors	Finding out which **prime numbers multiply** together to make the **original** number. Use a **prime factor tree.** Also known as 'prime factorisation'.	

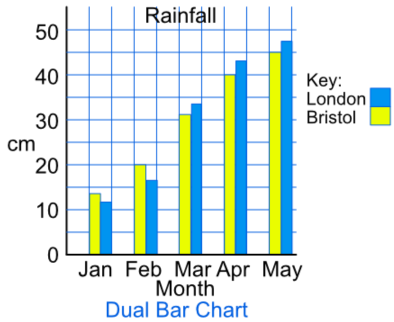
Topic: Representing Data

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Frequency Table</td><td>A record of how often each value in a set of data occurs.</td><td>
<table>
<tr><th>Number of marks</th><th>Tally marks</th><th>Frequency</th></tr>
<tr><td>1</td><td>卌 II</td><td>7</td></tr>
<tr><td>2</td><td>卌</td><td>5</td></tr>
<tr><td>3</td><td>卌 I</td><td>6</td></tr>
<tr><td>4</td><td>卌</td><td>5</td></tr>
<tr><td>5</td><td>III</td><td>3</td></tr>
<tr><td>Total</td><td></td><td>26</td></tr>
</table>
</td></tr>
<tr><td>2. Bar Chart</td><td>Represents data as vertical blocks.

$x - axis$ shows the type of data
$y - axis$ shows the frequency for each type of data
Each bar should be the same width
There should be gaps between each bar
Remember to label each axis.</td><td>


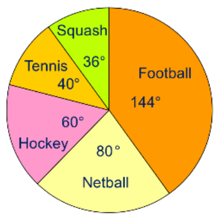
</td></tr>
<tr><td>3. Types of Bar Chart</td><td>Compound/Composite Bar Charts show data stacked on top of each other.

Comparative/Dual Bar Charts show data side by side.</td><td>




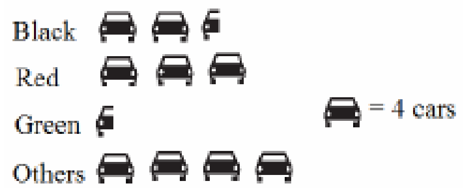
</td></tr>
<tr><td>4. Pie Chart</td><td>Used for showing how data breaks down into its constituent parts.

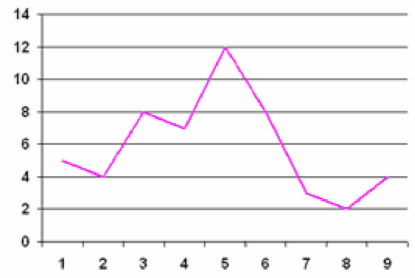
When drawing a pie chart, divide 360° by the total frequency. This will tell you how many degrees to use for the frequency of each category.

Remember to label the category that each sector in the pie chart represents.</td><td>


If there are 40 people in a survey, then each person will be worth 360÷40=9° of the pie chart.
</td></tr>
</table>

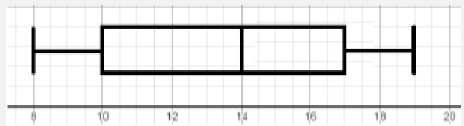
<table>
<tr><td>5. Pictogram</td><td>Uses pictures or symbols to show the value of the data.

A pictogram must have a key.</td><td>
</td></tr>
<tr><td>6. Line Graph</td><td>A graph that uses points connected by straight lines to show how data changes in values.

This can be used for time series data, which is a series of data points spaced over uniform time intervals in time order.</td><td>
</td></tr>
<tr><td>7. Two Way Tables</td><td>A table that organises data around two categories.

Fill out the information step by step using the information given.

Make sure all the totals add up for all columns and rows.</td><td>Question: Complete the 2 way table below.
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td></td><td>58</td></tr>
<tr><td>Girls</td><td></td><td></td><td></td></tr>
<tr><td>Total</td><td></td><td>84</td><td>100</td></tr>
</table>
Answer: Step 1, fill out the easy parts (the totals)
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td>48</td><td>58</td></tr>
<tr><td>Girls</td><td></td><td></td><td>42</td></tr>
<tr><td>Total</td><td>16</td><td>84</td><td>100</td></tr>
</table>
Answer: Step 2, fill out the remaining parts
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td>48</td><td>58</td></tr>
<tr><td>Girls</td><td>6</td><td>36</td><td>42</td></tr>
<tr><td>Total</td><td>16</td><td>84</td><td>100</td></tr>
</table></td></tr>
<tr><td>8. Box Plots</td><td>The minimum, lower quartile, median, upper quartile and maximum are shown on a box plot.

A box plot can be drawn independently or from a cumulative frequency diagram.</td><td>Students sit a maths test. The highest score is 19, the lowest score is 8, the median is 14, the lower quartile is 10 and the upper quartile is 17. Draw a box plot to represent this information.

</td></tr>
<tr><td>9. Comparing Box Plots</td><td>Write two sentences.
1. Compare the averages using the medians for two sets of data.
2. Compare the spread of the data using the range or IQR for two sets of data.

The <u>smaller</u> the range/IQR, the <u>more consistent</u> the data.

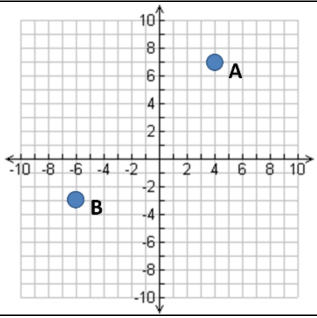
You must compare box plots in the context of the problem.</td><td>‘On average, students in class A were more successful on the test than class B because their median score was higher.’

‘Students in class B were more consistent than class A in their test scores as their IQR was smaller.’</td></tr>
</table>

Topic: Indices

Topic/Skill	Definition/Tips	Example
1. Square Number	The number you get when you **multiply a number by itself**.	**1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225…** $9^2 = 9 \times 9 = 81$
2. Square Root	The **number you multiply by itself** to get another number. The reverse process of squaring a number.	$\sqrt{36} = 6$ because $6 \times 6 = 36$
3. Solutions to $x^2 =$ ….	**Equations** involving **squares** have **two solutions**, one **positive** and one **negative**.	Solve $x^2 = 25$ $x = 5 \; or \; x = -5$ This can also be written as $x = \pm 5$
4. Cube Number	The number you get when you **multiply a number by itself and itself again**.	1, 8, 27, 64, 125… $2^3 = 2 \times 2 \times 2 = 8$
5. Cube Root	The **number you multiply by itself and itself again** to get another number. The reverse process of cubing a number.	$\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 125$
6. Powers of…	The powers of a number are that **number raised to various powers**.	The powers of 3 are: $3^1 = 3$ $3^2 = 9$ $3^3 = 27$ $3^4 = 81$ etc.
7. Multiplication Index Law	When **multiplying** with the same base (number or letter), **add the powers**. $\boldsymbol{a^m \times a^n = a^{m+n}}$	$7^5 \times 7^3 = 7^8$ $a^{12} \times a = a^{13}$ $4x^5 \times 2x^8 = 8x^{13}$
8. Division Index Law	When **dividing** with the same base (number or letter), **subtract the powers**. $\boldsymbol{a^m \div a^n = a^{m-n}}$	$15^7 \div 15^4 = 15^3$ $x^9 \div x^2 = x^7$ $20a^{11} \div 5a^3 = 4a^8$
9. Brackets Index Laws	When raising a power to another power, multiply the powers together. $\boldsymbol{(a^m)^n = a^{mn}}$	$(y^2)^5 = y^{10}$ $(6^3)^4 = 6^{12}$ $(5x^6)^3 = 125x^{18}$
10. Notable Powers	$\boldsymbol{p = p^1}$ $\boldsymbol{p^0 = 1}$	$99999^0 = 1$
11. Negative Powers	A negative power performs the reciprocal. $\boldsymbol{a^{-m} = \frac{1}{a^m}}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
12. Fractional Powers	The denominator of a fractional power acts as a 'root'. The numerator of a fractional power acts as a normal power. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$	$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$ $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{25}}{\sqrt{16}}\right)^3 = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$

Topic: Coordinates and Linear Graphs

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Coordinates</td><td>Written in pairs. The first term is the x-coordinate (movement across). The second term is the y-coordinate (movement up or down)</td><td>

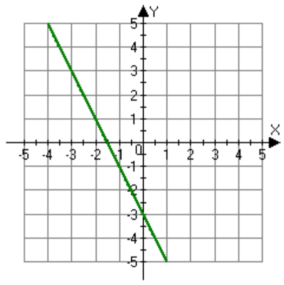
A: (4,7)
B: (-6,-3)</td></tr>
<tr><td>2. Midpoint of a Line</td><td>Method 1: add the x coordinates and divide by 2, add the y coordinates and divide by 2

Method 2: Sketch the line and find the values half way between the two x and two y values.</td><td>Find the midpoint between (2,1) and (6,9)

$\frac{2+6}{2} = 4$ and $\frac{1+9}{2} = 5$

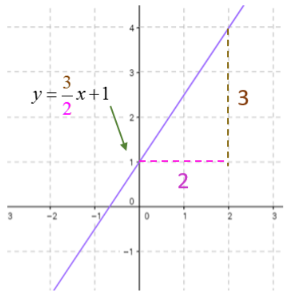
So, the midpoint is (4,5)</td></tr>
<tr><td>3. Linear Graph</td><td>Straight line graph.

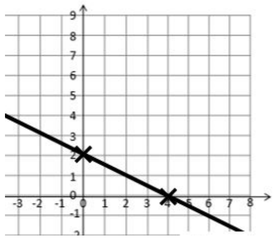
The general equation of a linear graph is
$$\boldsymbol{y = mx + c}$$
where $\boldsymbol{m}$ is the gradient and c is the y-intercept.

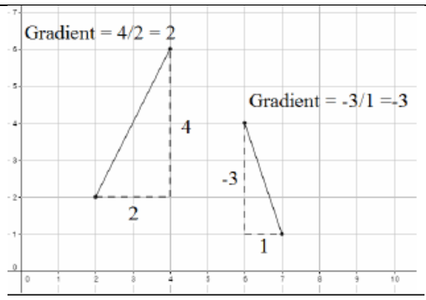
The equation of a linear graph can contain an x-term, a y-term and a number.</td><td>Example:


Other examples:
$x = y$
$y = 4$
$x = -2$
$y = 2x - 7$
$y + x = 10$
$2y - 4x = 12$</td></tr>
<tr><td>4. Plotting Linear Graphs</td><td>Method 1: Table of Values
Construct a table of values to calculate coordinates.

Method 2: Gradient-Intercept Method (use when the equation is in the form $y = mx + c$)
1. Plots the y-intercept
2. Using the gradient, plot a second point.
3. Draw a line through the two points plotted.

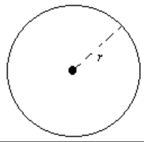
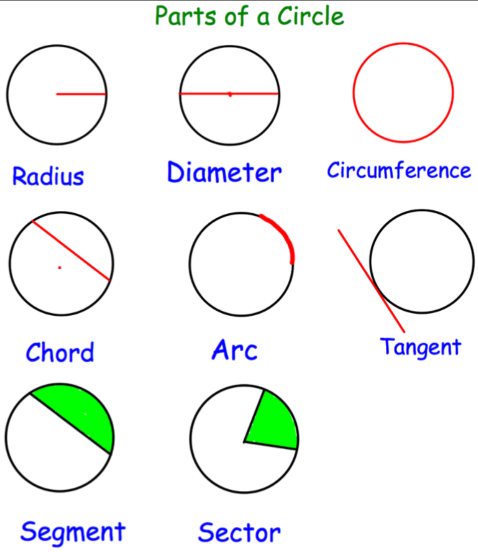
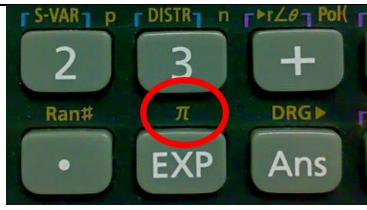
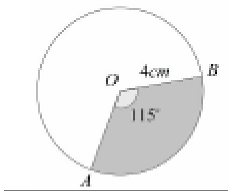
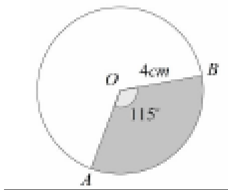
Method 3: Cover-Up Method (use when the equation is in the form $ax + by = c$)
1. Cover the x term and solve the resulting equation. Plot this on the $x - axis$.
2. Cover the y term and solve the resulting equation. Plot this on the $y - axis$.
3. Draw a line through the two points plotted.</td><td><table><tr><td>x</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y= x +3</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>


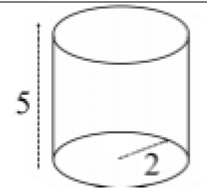
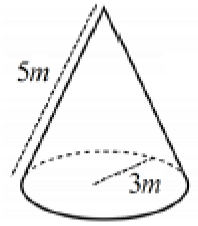

$2x + 4y = 8$</td></tr>
</table>

5. Gradient	The gradient of a line is how **steep** it is. **Gradient =** $$\frac{\boldsymbol{Change\ in\ y}}{\boldsymbol{Change\ in\ x}} = \frac{\boldsymbol{Rise}}{\boldsymbol{Run}}$$ The gradient can be positive (sloping upwards) or negative (sloping downwards)	
6. Finding the Equation of a Line <u>given a point and a gradient</u>	**Substitute** in the **gradient (m)** and **point (x,y)** in to the equation $\boldsymbol{y = mx + c}$ and **solve for c**.	Find the equation of the line with gradient 4 passing through (2,7). $y = mx + c$ $7 = 4 \times 2 + c$ $c = -1$ $y = 4x - 1$
7. Finding the Equation of a Line <u>given two points</u>	Use the two points to **calculate the gradient**. Then **repeat the method above** using the gradient and either of the points.	Find the equation of the line passing through (6,11) and (2,3) $m = \frac{11 - 3}{6 - 2} = 2$ $y = mx + c$ $11 = 2 \times 6 + c$ $c = -1$ $y = 2x - 1$
8. Parallel Lines	If two lines are **parallel**, they will have the **same gradient**. The value of m will be the same for both lines.	Are the lines $y = 3x - 1$ and $2y - 6x + 10 = 0$ parallel? Answer: Rearrange the second equation in to the form $y = mx + c$ $2y - 6x + 10 = 0 \rightarrow y = 3x - 5$ Since the two gradients are equal (3), the lines are parallel.
9. Perpendicular Lines	If two lines are **perpendicular**, the **product** of their **gradients** will always equal **-1**. The gradient of one line will be the **negative reciprocal** of the gradient of the other line. You may need to rearrange equations of lines to compare gradients (they need to be in the form $y = mx + c$)	Find the equation of the line perpendicular to $y = 3x + 2$ which passes through (6,5) Answer: As they are perpendicular, the gradient of the new line will be $-\frac{1}{3}$ as this is the negative reciprocal of 3. $y = mx + c$

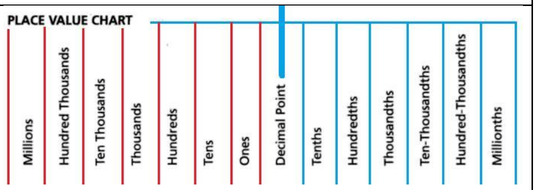
		$$5 = -\frac{1}{3} \times 6 + c$$ $$c = 7$$ $$y = -\frac{1}{3}x + 7$$ Or $$3x + x - 7 = 0$$

Topic: Circumference and Area

Topic/Skill	Definition/Tips	Example
1. Circle	A circle is the locus of all points equidistant from a central point.	
2. Parts of a Circle	**Radius** – the **distance** from the **centre** of a circle to the **edge** **Diameter** – the total **distance** across the **width** of a circle **through the centre**. **Circumference** – the **total distance** around the **outside** of a circle **Chord** – a **straight line** whose **end points lie on a circle** **Tangent** – a **straight line** which **touches** a circle at exactly **one point** **Arc** – a **part of the circumference** of a circle **Sector** – the **region** of a circle enclosed by **two radii** and their intercepted **arc** **Segment** – the **region** bounded by a **chord** and the **arc** created by the chord	
3. Area of a Circle	$\boldsymbol{A = \pi r^2}$ which means 'pi x radius squared'.	If the radius was 5cm, then: $A = \pi \times 5^2 = 78.5cm^2$
4. Circumference of a Circle	$\boldsymbol{C = \pi d}$ which means 'pi x diameter'	If the radius was 5cm, then: $C = \pi \times 10 = 31.4cm$
5. π ('pi')	Pi is the circumference of a circle divided by the diameter. $\boldsymbol{\pi \approx 3.14}$	
6. Arc Length of a Sector	The arc length is part of the circumference. Take the **angle** given **as a fraction over 360°** and **multiply** by the **circumference**.	Arc Length = $\frac{115}{360} \times \pi \times 8 = 8.03cm$ 
7. Area of a Sector	The area of a sector is part of the total area. Take the **angle** given **as a fraction over 360°** and **multiply** by the **area**.	Area = $\frac{115}{360} \times \pi \times 4^2 = 16.1cm^2$ 

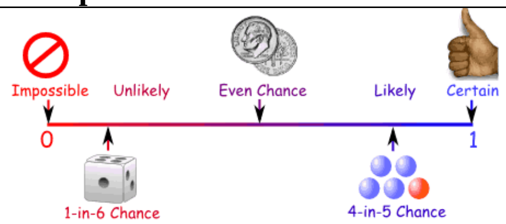
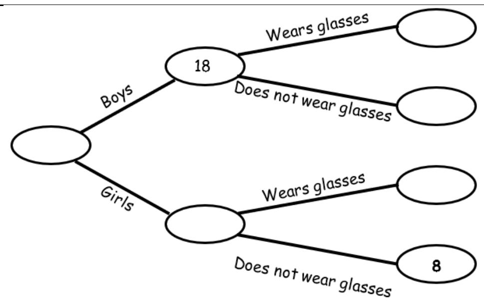
8. Surface Area of a Cylinder	**Curved Surface Area =** $\boldsymbol{\pi dh}$ or $\boldsymbol{2\pi rh}$ **Total SA =** $\boldsymbol{2\pi r^2 + \pi dh}$ or $\boldsymbol{2\pi r^2 + 2\pi rh}$	 $Total\ SA = 2\pi(2)^2 + \pi(4)(5) = 28\pi$
9. Surface Area of a Cone	**Curved Surface Area =** $\boldsymbol{\pi rl}$ where $l = slant\ height$ **Total SA =** $\boldsymbol{\pi rl + \pi r^2}$ You may need to use Pythagoras' Theorem to find the slant height	 $Total\ SA = \pi(3)(5) + \pi(3)^2 = 24\pi$
10. Surface Area of a Sphere	$\boldsymbol{SA = 4\pi r^2}$ Look out for hemispheres – halve the SA of a sphere and add on a circle (πr^2)	Find the surface area of a sphere with radius 3cm. $SA = 4\pi(3)^2 = 36\pi cm^2$

Topic: Accuracy

Topic/Skill	Definition/Tips	Example
1. Place Value	The **value** of where a **digit** is within a number.	In 726, the value of the 2 is 20, as it is in the ‘tens’ column.
2. Place Value Columns	The names of the columns that **determine the value of each digit**. The ‘ones’ column is also known as the ‘units’ column.	
3. Rounding	To make a number simpler but keep its value close to what it was. If the **digit to the right** of the rounding digit is **less than 5, round down**. If the **digit to the right** of the rounding digit is **5 or more, round up**.	74 rounded to the nearest ten is 70, because 74 is closer to 70 than 80. 152,879 rounded to the nearest thousand is 153,000.
4. Decimal Place	The **position** of a digit to the **right of a decimal point**.	In the number 0.372, the 7 is in the second decimal place. 0.372 rounded to two decimal places is 0.37, because the 2 tells us to round down. Careful with money - don’t write £27.4, instead write £27.40
5. Significant Figure	The significant figures of a number are the digits which **carry meaning** (ie. are significant) to the size of the number. The **first significant figure** of a number **cannot be zero**. In a number with a decimal, trailing zeros are not significant.	In the number 0.00821, the first significant figure is the 8. In the number 2.740, the 0 is not a significant figure. 0.00821 rounded to 2 significant figures is 0.0082. 19357 rounded to 3 significant figures is 19400. We need to include the two zeros at the end to keep the digits in the same place value columns.
6. Truncation	A method of approximating a decimal number by **dropping all decimal places** past a certain point **without rounding**.	3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416)
7. Error Interval	A **range of values** that a number could have taken before being rounded or truncated. An error interval is written using inequalities, with a **lower bound** and an **upper bound**.	0.6 has been rounded to 1 decimal place. The error interval is: $0.55 \leq x < 0.65$ The lower bound is 0.55 The upper bound is 0.65

	Note that the lower bound inequality can be 'equal to', but the upper bound cannot be 'equal to'.	
8. Estimate	To find something **close to the correct answer**.	An estimate for the height of a man is 1.8 metres.
9. Approximation	When using approximations to estimate the solution to a calculation, **round each number in the calculation to 1 significant figure**. ≈ means 'approximately equal to'	$\frac{348+692}{0.526} \approx \frac{300+700}{0.5} = 2000$ 'Note that dividing by 0.5 is the same as multiplying by 2'
10. Rational Number	A number of the form $\boldsymbol{\frac{p}{q}}$**, where $\boldsymbol{p}$ and $\boldsymbol{q}$ are integers** and $\boldsymbol{q \neq 0}$. A number that cannot be written in this form is called an 'irrational' number	$\frac{4}{9}, 6, -\frac{1}{3}, \sqrt{25}$ are examples of rational numbers. $\pi, \sqrt{2}$ are examples of an irrational numbers.
11. Surd	The **irrational number** that is a **root of a positive integer,** whose value cannot be determined exactly. Surds have **infinite non-recurring decimals**.	$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly. $\sqrt{2} = 1.41421356\ldots$ which never repeats.
12. Rules of Surds	$\boldsymbol{\sqrt{ab} = \sqrt{a} \times \sqrt{b}}$ $\boldsymbol{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$ $\boldsymbol{a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}}$ $\boldsymbol{\sqrt{a} \times \sqrt{a} = a}$	$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$ $2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$ $\sqrt{7} \times \sqrt{7} = 7$
13. Rationalise a Denominator	The process of rewriting a fraction so that the **denominator contains only rational numbers**.	$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6}}{2}$ $\frac{6}{3+\sqrt{7}} = \frac{6(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$ $= \frac{18-6\sqrt{7}}{9-7}$ $= \frac{18-6\sqrt{7}}{2} = 9 - 3\sqrt{7}$

Topic: Basic Probability

Topic/Skill	Definition/Tips	Example
1. Probability	The **likelihood/chance** of something happening. Is expressed as a number **between 0 (impossible) and 1 (certain)**. Can be expressed as a fraction, decimal, percentage or in words (likely, unlikely, even chance etc.)	
2. Probability Notation	**P(A)** refers to the **probability that event A will occur**.	P(Red Queen) refers to the probability of picking a Red Queen from a pack of cards.
3. Theoretical Probability	$\frac{\textbf{Number of Favourable Outcomes}}{\textbf{Total Number of Possible Outcomes}}$	Probability of rolling a 4 on a fair 6-sided die = $\frac{1}{6}$.
4. Relative Frequency	$\frac{\textbf{Number of Successful Trials}}{\textbf{Total Number of Trials}}$	A coin is flipped 50 times and lands on Tails 29 times. The relative frequency of getting Tails = $\frac{29}{50}$.
5. Expected Outcomes	To find the number of expected outcomes, **multiply** the **probability** by the **number of trials**.	The probability that a football team wins is 0.2 How many games would you expect them to win out of 40? $0.2 \times 40 = 8\ games$
6. Exhaustive	Outcomes are **exhaustive** if they **cover the entire range of possible outcomes**. The **probabilities** of an **exhaustive** set of outcomes **adds up to 1**.	When rolling a six-sided die, the outcomes 1, 2, 3, 4, 5 and 6 are exhaustive, because they cover all the possible outcomes.
7. Mutually Exclusive	Events are mutually exclusive if they **cannot happen at the same time**. The **probabilities** of an exhaustive set of **mutually exclusive** events **adds up to 1**.	Examples of mutually exclusive events: - Turning left and right - Heads and Tails on a coin Examples of non mutually exclusive events: - King and Hearts from a deck of cards, because you can pick the King of Hearts
8. Frequency Tree	A diagram showing how information is categorised into various categories. The **numbers** at the ends of branches tells us how often something happened (**frequency**).	

<table>
<tr><td></td><td>The lines connected the numbers are called branches.</td><td></td></tr>
<tr><td>9. Sample Space</td><td>The set of all possible outcomes of an experiment.</td><td>
<table>
<tr><th>+</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
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<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
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</td></tr>
<tr><td>10. Sample</td><td>A sample is a small selection of items from a population.

A sample is biased if individuals or groups from the population are not represented in the sample.</td><td>A sample could be selecting 10 students from a year group at school.</td></tr>
<tr><td>11. Sample Size</td><td>The larger a sample size, the closer those probabilities will be to the true probability.</td><td>A sample size of 100 gives a more reliable result than a sample size of 10.</td></tr>
</table>

Topic: Fractions

Topic/Skill	Definition/Tips	Example
1. Fraction	A mathematical expression representing the **division** of one integer by another. Fractions are written as **two numbers separated by a horizontal line**.	$\frac{2}{7}$ is a 'proper' fraction. $\frac{9}{4}$ is an 'improper' or 'top-heavy' fraction.
2. Numerator	The **top** number of a fraction.	In the fraction $\frac{3}{5}$, 3 is the numerator.
3. Denominator	The **bottom** number of a fraction.	In the fraction $\frac{3}{5}$, 5 is the denominator.
4. Unit Fraction	A fraction where the **numerator is one** and the denominator is a positive integer.	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\ etc.$ are examples of unit fractions.
5. Reciprocal	The reciprocal of a number is **1 divided by the number**. The reciprocal of x is $\frac{1}{x}$ **When we multiply a number by its reciprocal we get 1**. This is called the 'multiplicative inverse'.	The reciprocal of 5 is $\frac{1}{5}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $\frac{2}{3} \times \frac{3}{2} = 1$
6. Mixed Number	A number formed of both an **integer part** and a **fraction part**.	$3\frac{2}{5}$ is an example of a mixed number.
7. Simplifying Fractions	**Divide the numerator and denominator by the highest common factor**.	$\frac{20}{45} = \frac{4}{9}$
8. Equivalent Fractions	Fractions which represent the **same value**.	$\frac{2}{5} = \frac{4}{10} = \frac{20}{50} = \frac{60}{150}\ etc.$
9. Comparing Fractions	To compare fractions, they each need to be rewritten so that they have a **common denominator**. **Ascending** means **smallest to biggest**. **Descending** means **biggest to smallest**.	Put in to ascending order : $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$. Equivalent: $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ Correct order: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
10. Fraction of an Amount	**Divide** by the **bottom**, **times** by the **top**	Find $\frac{2}{5}$ of £60 $60 \div 5 = 12$ $12 \times 2 = 24$
11. Adding or Subtracting Fractions	Find the **LCM of the denominators** to find a common denominator. Use equivalent fractions to change each fraction to the **common denominator**.	$\frac{2}{3} + \frac{4}{5}$ Multiples of 3: 3, 6, 9, 12, **15**.. Multiples of 5: 5, 10, **15**.. LCM of 3 and 5 = 15

	Then just **add or subtract the numerators** and keep the **denominator the same**.	$\frac{2}{3}=\frac{10}{15}$ $\frac{4}{5}=\frac{12}{15}$ $\frac{10}{15}+\frac{12}{15}=\frac{22}{15}=1\frac{7}{15}$
12. Multiplying Fractions	**Multiply** the **numerators** together and **multiply** the **denominators** together.	$\frac{3}{8}\times\frac{2}{9}=\frac{6}{72}=\frac{1}{12}$
13. Dividing Fractions	**‘Keep it, Flip it, Change it – KFC’** Keep the first fraction the same Flip the second fraction upside down Change the divide to a multiply Multiply by the reciprocal of the second fraction.	$\frac{3}{4}\div\frac{5}{6}=\frac{3}{4}\times\frac{6}{5}=\frac{18}{20}=\frac{9}{10}$

Topic: Basic Percentages

Topic/Skill	Definition/Tips	Example
1. Percentage	**Number of parts per 100.**	31% means $\frac{31}{100}$
2. Finding 10%	To find **10%**, **divide by 10**	10% of £36 = 36÷10=£3.60
3. Finding 1%	To find **1%**, **divide by 100**	1% of £8 = 8÷100 = £0.08
4. Percentage Change	$\frac{\boldsymbol{Difference}}{\boldsymbol{Original}} \times \mathbf{100}\%$	A games console is bought for £200 and sold for £250. % change = $\frac{50}{200} \times 100 = 25\%$
5. Fractions to Decimals	**Divide the numerator by the denominator** using the bus stop method.	$\frac{3}{8} = 3 \div 8 = 0.375$
6. Decimals to Fractions	**Write as a fraction** over 10, 100 or 1000 and simplify.	$0.36 = \frac{36}{100} = \frac{9}{25}$
7. Percentages to Decimals	**Divide by 100**	$8\% = 8 \div 100 = 0.08$
8. Decimals to Percentages	**Multiply by 100**	$0.4 = 0.4 \times 100\% = 40\%$
9. Fractions to Percentages	Percentage is just a fraction out of 100. **Make the denominator 100 using equivalent fractions**. When the denominator doesn't go in to 100, use a calculator and **multiply the fraction by 100**.	$\frac{3}{25} = \frac{12}{100} = 12\%$ $\frac{9}{17} \times 100 = 52.9\%$
10. Percentages to Fractions	Percentage is just a fraction out of 100. **Write the percentage over 100** and simplify.	$14\% = \frac{14}{100} = \frac{7}{50}$

Topic: Calculating with Percentages

Topic/Skill	Definition/Tips	Example
1. Increase or Decrease by a Percentage	Non-calculator: **Find the percentage** and **add** or **subtract** it from the **original** amount. Calculator: Find the **percentage multiplier** and multiply.	<u>Increase 500 by 20% (Non Calc):</u> 10% of 500 = 50 so 20% of 500 = 100 500 + 100 = 600 <u>Decrease 800 by 17% (Calc):</u> 100%-17%=83% 83% ÷ 100 = 0.83 0.83 x 800 = 664
2. Percentage Multiplier	The **number** you **multiply** a quantity by to **increase or decrease** it by a **percentage**.	The multiplier for increasing by 12% is 1.12 The multiplier for decreasing by 12% is 0.88 The multiplier for increasing by 100% is 2.
3. Reverse Percentage	Find the **correct percentage given in the question**, then work backwards to **find 100%** Look out for words like '**before**' or '**original**'	A jumper was priced at £48.60 after a 10% reduction. Find its original price. 100% - 10% = 90% 90% = £48.60 1% = £0.54 100% = £54
4. Simple Interest	Interest calculated as a **percentage of the original** amount.	£1000 invested for 3 years at 10% simple interest. 10% of £1000 = £100 Interest = $3 \times £100 = £300$

Topic: Algebra

Topic/Skill	Definition/Tips	Example
1. Expression	A mathematical statement written using **symbols**, **numbers** or **letters**,	3x + 2 or $5y^2$
2. Equation	A statement showing that **two expressions are equal**	2y – 17 = 15
3. Identity	An equation that is **true for all values** of the variables An identity uses the symbol: ≡	$2x \equiv x+x$
4. Formula	Shows the **relationship** between **two or more variables**	Area of a rectangle = length x width or A= LxW
5. Simplifying Expressions	**Collect 'like terms'.** Be careful with negatives. x^2 and x are not like terms.	$2x+3y+4x-5y+3$ $=6x-2y+3$ $3x+4-x^2+2x-1=5x-x^2+3$
6. x times x	The answer is x^2 not $2x$.	Squaring is multiplying by itself, not by 2.
7. $p \times p \times p$	The answer is p^3 not $3p$	If p=2, then p^3=2x2x2=8, not 2x3=6
8. $p+p+p$	The answer is 3p not p^3	If p=2, then 2+2+2=6, not $2^3=8$
9. Expand	To expand a bracket, **multiply** each term **in the bracket** by the expression **outside** the bracket.	$3(m+7)=3x+21$
10. Factorise	The **reverse** of **expanding**. Factorising is writing an expression as a product of terms by **'taking out' a common factor**.	$6x-15=3(2x-5)$, where 3 is the common factor.